Before I was a mathematician

The pursuit of a pure mathematician is as much a journey through the interior life as it is a practice. This life is often quite different from that of other scientists: pure mathematicians don't have labs or employees, we don't go on expeditions, or try to save the world through our research. We collaborate with other mathematicians and try to communicate our strange and esoteric world to others, but we are driven by other forces not so easy to describe. There is no rigid hierarchical structure among mathematicians, so the struggles we face in our careers are perhaps more personal than for other scientists.

The low number of women in mathematical careers is often blamed on a pipeline problem: not enough women majoring in math leads to fewer applicants to PhD programs. The ranks of women are further thinned by societal sexism, two-body problems, lack of childcare, and so on. I think there is much truth to this hypothesis, and that the pipeline issue begins in early childhood. We need to address the attrition in math of girls and other underrepresented groups at an early stage in life.

I am writing this as a senior mathematician, a tenured full professor at the University of Chicago. I think that my story tells some kind of tale of survival, and I've chosen to focus here on my early years, long before I started my PhD and properly began my career. Part of the outcome I owe to special circumstances and advantages I've enjoyed, while other parts are more universal, or at least I imagine could be shared by a person in a very different life or in a far away place.

I was born in the late 60s. The Vietnam War and Watergate carried significant weight in my parents' lives, and in mine as well. My parents took me campaigning door to door for George McGovern and talked to me frankly about the state of our country and the world. It was the early days of civil rights legislation and women's rights had begun to find a public voice. The kids I knew felt empowered and liberated: we had Sesame Street, the Electric Company and Zoom on public television and Free to Be You and Me on our record players.

I was a curious child, in more than one sense of the word. I constantly got into things, made messes, started fires, broke things and then tried to put them back together so that no one would notice, fed my grandparents' dogs the contents of their medicine cabinet and fed my little sister a box of matches (the tips, at least). I started baking at the age of six, and soon made a peach pie without the help or knowledge of my parents. A few years later they found me alone in the kitchen attempting to light a heavily liquored "oranges flambé" and were aghast. My life was filled with experiments, many of them failures, like when I attempted to gather a bottle of honey from honeysuckle flowers.

I was largely indifferent to typical childhood activities, especially ones that required social interaction. Ten years ago, not long after my mother died, I came to suspect that she was on the autism spectrum. No doubt my father was neurodivergent as well, but for some men that gets

wrapped into one's persona as a scientist and goes unnoticed. My behavior as a child points in that direction, and my own son was similar when he was young. My difficulties are slight, and I've learned to accommodate them, but reading other people on the spot has never come easily to me, and I still spend a lot of time in my own thoughts.

When I was very small, I'd spend days in the garden with our next door neighbor, who explained to me earthworms and how they aerate the soil. In kindergarten I recall spending the entire recess period hanging out in the bushes playing with katydids, which are those insects that look like leaves. In fifth grade, I learned how to escape the playground through a hole in the fence and wandered the surrounding neighborhood during recess, usually with a friend in tow, igniting dandelions with a lighter we found and getting yelled at by random adults. I chafed at the powerlessness of childhood and rebelled, but in a sneaky way that would catch adults by surprise.

My mother had a conspiratorial nature and a wild imagination. She was an early feminist who was caught by surprise by two children within three years of getting married. After spending a few very frustrating years as a stay-at-home mom, she decided to take back her maiden name when I was 4 and my sister 2 and applied to law school. She joined NOW and worked toward passage of the ERA. She also taught me feminism with a distinctly misandrist bent. Once I called another boy in preschool a male chauvinist pig; his horrified mother reported this to my mom, who congratulated me. I admired my mother and emulated her and was proud that she had a different last name. She gave me the notion that I could do anything I wanted and that I would probably have to fight like hell to do it.

My father was a scientist, with a keen eye for the playful. While my mother was in law school, he was a Ph.D. student who assumed a lot of childcare responsibilities. For my sister and me, this meant hanging around with a slightly overgrown child who was most engaged with us when he himself was engaged in a project. With us, my dad built a giant kite almost twice his height out of lumber and shirting material, organized craft projects like making Valentine's Day sculptures out of dough for my mom, went sledding down a giant hill, built an elaborate haunted house in the basement for the neighborhood kids, and took us to the computer center to wander idly while he worked and then went home to build card houses with data entry cards. Like my mom, my dad resisted conventional constraints. A high school math whiz, he chose to major in English in college. He next went to divinity school where he met my mom, and then decided to become a social worker. Halfway through a PhD program in psychology, he became fascinated with computers and the statistical side of the discipline, a field called psychometrics. He dedicated his dissertation to his great loves: his wife, his children, and the computer. He switched careers several times in his life, taught himself both Hebrew and Russian, commuted on rollerskates, and was an accomplished amateur pianist. In the late 70s he built himself a computer — a Cromemco Z-2 — from a kit, the first of many computers in our house, on which my sister and I learned to program in Basic.

It was a chaotic upbringing in many ways. My parents were usually engrossed in their own activities, so my sister and I were frequently left alone. One morning in third grade, I was unable to lock the door before heading off to school, so I dialed 911 and got a ride to school in a police sedan. Another morning, I opened the door to discover that it was warmer than expected, so I found a pair of scissors and cut my jeans into shorts. When I arrived at school, my teacher took me aside and gave me her lab coat to wear. I had cut the pants in back so short you could see my underwear. Besides getting into trouble, I read a lot of books, mostly mysteries like the Nancy Drew series. I preferred the company of adults and was indulged by teachers. I did well in school but occasionally would get report cards with things like "inconsistent!" written on them. On the whole, I would use words such as edenic, pure, and unconstrained to describe my early childhood.

Looking back, I think the freedom in my early years allowed my scientific side to develop, but of course curiosity can be channeled in many directions. My sister, two years younger than I, is now a writer and poet. I might have become an artist. My grandparents, who had worked in magazine publishing, retired to a beach community in the late 70s and bought an art gallery that they ran until the 80's. I spent hours in that gallery with the run of the place every summer. My grandfather was a watercolor artist, and I have some artistic skill, so we connected on that level. He shared his techniques and taste and gave unvarnished advice. When I was twelve, I told him I might want to be an artist, and he told me to carry out the first lesson he was given in design school: to draw a chair. He tore apart the result and taught me the secret of drawing chairs that wouldn't fall over in real life. I was hooked. As it turns out, my parents had different plans for me.

My parents pushed for me academically behind the scenes, but I was unaware of it. I discovered a few days before starting fourth grade that I was to skip fourth grade (the news had been on my report card that spring, but I hadn't been able to collect it due to unpaid library fines). I found out later that my parents had been asking schools for some time whether I could move ahead, receiving answers like "she can't move ahead. She doesn't know the difference between a rectangle and a triangle" (when I was 6). I don't think I ever complained of being bored in school, but I was competitive and challenged my teachers. As an older child I discovered I'd been given an IQ test when I was younger. My parents probably saw things in me that I couldn't have. At some point, they saw my affinity for math, and I think they took great pride themselves in it. I rarely felt pressure from them, for which I am grateful.

By the time I entered high school, math held a special place in my life. Partly this was because I did well in math at school, and I was rewarded by adults. I had also gotten hooked on the adrenaline rush from solving problems quickly, and I had managed to skip ahead enough that I took calculus my sophomore year, when I was 14. I found a social niche on the school math team with a rowdy bunch of misfits who were fond of rock music and drinking too much booze. I pursued other social options as well, hanging out increasingly with an artsy crowd that was fond

of alternative music and espresso. I strutted around like I didn't care, but I did enough work to excel in school, and my grades were important to me.

I was definitely aware that my talent for, and interest in, math and science was unusual for a girl. It became my persona, and a point of pride, and I saw it as my duty to leave as many boys in the dust as possible. There was another kid on math team who was very good and viewed as my rival, maybe even my better. I kind of despised him: he was loud and arrogant and a know-it-all and seemed to think he was above the rules. He and I were stuck together one year at the state math team finals as our school's two-person team, in a type of speed competition with an audience. We came in second place, but we battled each other as much as we did the other teams. He was faster, but I was more accurate. I still remember one problem he figured out almost instantly, which impressed me at the time: *the side lengths of a square are increased by 20%. How much is the area increased?* Answer: 44%, since 12^2 = 144, and 144 -100 = 44. Our high school had two senior math prizes: one for a girl and the other an overall prize. It seemed obvious how those prizes would be split, but I ended up winning both. Afterwards, my rival approached me and told me he should have gotten "the prize for boys." I laughed at him for being so baldly sexist, even though I could feel his hurt. He did not end up a mathematician, but he did make a lot of money eventually.

Most people don't decide to be a mathematician on the basis of an even remotely accurate idea of what a mathematician is or does. Mathematical research bears very little resemblance to doing grade-school mathematics, and yet it's a career path that is normally chosen before adulthood. (There are exceptions, prodigies who get so advanced that they start to read real math books, usually under the guidance of an adult who knows some advanced mathematics.) Back in the 80s, there were plenty of math contests for teenagers in the US but very few activities that involved sustained exploration of mathematical concepts, like the math circles that have risen to prominence in the last 10 years. There also weren't that many mathematicians around to serve as role models, especially women or people of color. Having a scientist dad definitely gave me an advantage, a notion of what it might be to pursue a career in research. In contrast to my mom, who worked long hours as an associate in a law firm, my dad seemed to have a lot of time just to hang out at the computer and indulge his many hobbies.

When I applied to college, I knew my career path: a research mathematician. A couple of years ago, I retrieved my application to Harvard (where I ended up going) because I was curious about what I had written in the essays. In places, what I wrote smacks of a certain calculated insincerity, like this gem: "Occasionally, I sit down and reprove the Mean Value Theorem; it brings back happy memories" (which someone in the application office set apart in braces, perhaps taken with my apparent eccentricity bordering on fanaticism). Mostly, though, I seem to have been earnestly passionate about math. I wrote "I am certain that I will enter a scientific profession, and I am fairly sure that I will become a mathematician […] I want to make thinking my profession."

At Harvard, I did well in the advanced math class for freshmen, in part because I had seen much of the material before. Sometime in my second year, though, I got lost, at least mathematically. It started in the abstract algebra class, where we studied group theory, and specifically in the subject of cosets, which to my mind are one of the first truly symbolic things you encounter in mathematics. I could not find a way to visualize them. I now know that I was bringing the wrong mindset to the subject of group theory: whereas I was trying to abstractify numbers, I should have been exploring a new notion, that of *motion*. Groups, as I realized later, consist of *moves*, or actions. The study of moves is the origin of the subject, but a layer of abstraction had been added later in the natural process of generalization (an important process, I might add, as abstractification can give the clarity necessary to discover deeper phenomena). And yet the origins of the subject were lost to me, and the professor gave no indication of how to intuit the subject, or at least not that I remember.

The rest of my college years had ups and downs, and my focus on mathematics waned as I made friends and explored other subjects. My interest in art intensified, and I wrote two articles for a student art journal, one about Picasso's Guernica and the other about Dante Gabriel Rossetti and the wounded Amazon theme in Greek art. I took part in a summer math research program, where I proved my first theorem, about Hamiltonian circuits in finite cyclic groups. These were the same groups that had stumped me in the algebra class, but I was able to understand them in this context because they had been turned into geometric objects, called Cayley graphs. A finite cyclic group could be seen as living on the surface of a torus, which is a mathematical version of a doughnut. With this point of view, with the ability to draw pictures, I could see my way to an understanding and discover a new phenomenon. In the end, my paper did little more than reprove existing results but in a novel and simplified way. The paper was published in a journal and later played an instrumental role in my admission to graduate school.

That same summer, a graduate student mentor in the program I attended came to give a talk about a subject he studied: *ergodic theory*. The origins of the term "ergodic" are obscure, but the root "erg" is from Greek and means *work*. This student talked about a simple problem of calculating the average temperature of a circular ring of wire. There were two ways to go about this, he explained. The first is to measure the temperature at every point in the ring, a type of temperature snapshot, and take an average of these numbers all at once, through a process called integration. This is a *static* calculation: one observes the ring at a moment in time and then calculates the answer. By contrast, the second method, he explained, is *dynamic*: it involves exploring the ring *over time*. More precisely, starting at a fixed place in the ring, one hops to a new place a fixed angular distance from the starting point and records the temperature there. Repeating this process, one hops again and again to new places on the ring, each place a fixed angular distance from the previous one, and each time recording the temperature at that place. One can thus keep a running average of the temperatures recorded. The *ergodic theorem* then says that as long as that angular distance is chosen to be irrational (not a fractional multiple of 360 degrees), these running averages will converge over time to the static average. In short, "space averages = time averages." I loved this. A whole new dimension, a life, brought to math.

The subject of ergodic theory, and more generally its root field of *dynamical systems* came to fascinate me. There were no official courses on the subject at Harvard, but a graduate student taught a seminar one quarter, and I learned it from him. I learned about a graduate course at Boston University on ergodic theory, which I attended, although I was mostly lost. I decided to write a senior thesis on ergodic theory, and the same graduate student gave me a topic, a conjecture about entropy. I felt a bit like I was exploring the wilderness, with little official supervision by the faculty at Harvard, but I liked it. It gave room to do my own thing, as I didn't seem to be particularly great at doing what the faculty themselves did.

I had not applied to graduate schools because no one had encouraged me to do it, I had mediocre grades including a "C" in topology, and honestly I was disillusioned with what I viewed as "mainstream math" at that point. By the spring of my senior year, however, I started to think that maybe grad school was worth a try. I went to talk to the Director of Undergraduate Studies, the same professor who had taught me algebra two years earlier. At our meeting, I explained my tentative change of heart.

"If you do choose to pursue mathematics further," he said "you should know that in the end there is a great pyramid of mathematicians. A small number are at the very top, but the vast majority are at the bottom of this pyramid. As long as you're comfortable with the possibility of being at the bottom, I see no reason not to continue." For the three years at Harvard that followed my first year, I had received almost no encouragement, and then this. Fortunately, these words had the effect of pissing me off rather than discouraging me. I later learned that this professor had a knack for saying discouraging things to women undergraduates. He told one that she should drop the major after she struggled in a course (she went on to get a PhD and is a research mathematician). He told another, who had very good grades and had won a prize her honors thesis, that she should consider applying to midlevel graduate schools, where she'd have a chance of getting in (she went on to graduate school at Stanford and is now a member of the National Academy of Sciences and president of the American Mathematical Society). These women and I, and many other women at the time, learned that anger can be a great motivator.

After the graduation ceremony, I ran out of Harvard yard early with a couple of friends. We were stopped by a local tv news crew, who asked us what our plans were after college. "I'm working for an organization called Fairness and Accuracy in Reporting," said my friend. They turned to me. "I have no idea!" I said. I was the one who ended up on the evening news. I went home after that and sent out my resume. I got a job in actuarial consulting in downtown Chicago and found a tiny apartment. I loved the work, loved being able to streamline tasks for our clients, who were people working in the benefits office of large companies like Inland Steel. My boss told me he wanted to train me to replace him eventually. As much as enjoyed the work, I missed the math. I missed butting my head against something mysterious and the knowledge that there would always, always be something out there I did not already know. I applied to math PhD programs that fall and focused on my work.

To my genuine surprise, I was admitted to several graduate programs and was awarded an NSF graduate research fellowship. Despite my so-so grades, I had scored very well on the GRE's (thanks to my excellent high school education), and I had a good letter of reference. It came from the director of the undergraduate research program I had attended, a professor at the University of Minnesota in Duluth. He had shown me the letter, in which he included the editor's comments on my article that had been accepted to a combinatorics journal: "usually we require the reports of at least two referees, but this report was so strong I can accept this paper on the basis of only one." I visited graduate schools that spring and decided upon Berkeley, not just because it was the highest ranked school I had been admitted to, but because the campus was beautiful and there were dynamicists on the faculty.

Graduate school was a revelation. I was there because I wanted to do math. I had no other courses to distract me, and I was surrounded by people who loved math. And my attitude toward math had changed: it was no longer something I was doing to impress other people, and I no longer felt an obligation to mathematics as some type of calling. I couldn't picture myself as a professor, let alone as someone with a PhD, but I preferred it that way. My journey as a mathematician began there.