

Topological Topics. Ordinarily Wednesdays at 4 p.m. – (at most) 7 p.m. Room 308

This course is my version of Kazhdan’s “Basic Notions” seminar. Each week, I, or some other lecturer, will try to explain a basic topological idea, and what some of its implications are. This is the first of a multi-quarter course.

First Lectures (the outlines are largely for my benefit. The later the part, the more likely that they won’t be gotten to in the lecture, and might become IOUs)

1. Homology spheres: what do they look like and what are they good for.
  - i. Some examples in dimension three
    1. Poincare homology sphere via  $SU(2)$ , branched covers, isolated hypersurface singularities and via Eg.
    2. Digression on lens spaces.
    3. Dehn Lemma loop theorem, Sphere theorem and torus decompositions: hints about geometrization.
    4. Homology spheres by glueing knot complements. Some properties.
  2. Homology spheres continued
    - i. Inequivalent knots with the same complement?
      1. Inequivalent links
      2. Disk knots
    - ii. Knot companionship
      1. Connect summing
      2. (Twisted) Doubling
      3. Whitehead manifold
      4.  $Wh \times \mathbb{R} = \mathbb{R}^4$
      5. Geometrization for knot complements (a la Thurston)
    - iii. Dehn surgery
      1. Thurston’s theorem. Nonarithmeticity.
    - iv. Some examples in higher dimensions.
      1. Kervaire’s theorems.
      2. Mazur manifolds.
      3. Why dimension 4 is different
    - v. How they enter group actions.
3. Spectrum of the Laplacian (Starts at 5:15 because of Sarnak’s talk)
  - i. Definition of the Laplacian on manifolds and Regular graphs
  - ii. Calculation on tori, cycles, hypercubes.
  - iii. Cheeger’s inequality.
  - iv. Statement of Selberg’s theorem and a couple of words about Property T
  - v. Theorem of Folner-Kesten-Gerl-Brooks-Block-W
  - vi. Isospectral manifolds

1. Milnor's tori
  - a. From quadratic forms to modular forms
  - b. Riemann-Roch on an orbifold
  - c.  $Eg_k$
2. Sunada's method.
4. Adem Commutation varieties in compact Lie groups (Alejandro Adem – guest lecture 4-6 p.m.) + Department dinner (6:15 reception at the President's house followed by dinner at the Quad club)

### Topology and spaces of representations for abelian groups

Abstract: In this talk we describe basic topological properties for spaces of commuting elements in a compact Lie group, as well as their equivariant structure under conjugation. From this we derive information about their stable homotopy type and equivariant K-theory. This is joint work with Jose Gomez.

5. Homology spheres in high dimensions (cont'd).
  - i. Other applications of the proof of Kervaire's theorem to knot theory
  - ii. To counterexamples to the Smith conjecture in high dimensions
  - iii. Zeeman's examples (spinning and twist spinning)
  - iv. The h-cobordism theorem (first visit) and the Poincare conjecture
  - v. The + construction
  - vi. Connection of + construction to homology spheres.
  - vii. Homology spheres, logic and variational problems (if we're lucky)
  - viii. Connection to triangulability problems
  - ix. The Casson invariant

November 21 class is cancelled for Thanksgiving

6. This will be a short class, starting at 5:45 because of L.Silvestre's colloquium.