A DIFFICULTY IN THE CONCEPT OF SOCIAL WELFARE

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I. INTRODUCTION

In a capitalist democracy there are essentially two methods by which social choices can be made: voting, typically used to make "political" decisions, and the market mechanism, typically used to make "economic" decisions. In the emerging democracies with mixed economic systems, Great Britain, France, and Scandinavia, the same two modes of making social choices prevail, though more scope is given to the method of voting and to decisions based directly or indirectly on it and less to the rule of the price mechanism. Elsewhere in the world, and even in smaller social units within the democracies, the social decisions are sometimes made by single individuals or small groups and sometimes (more and more rarely in this modern world) by a widely encompassing set of traditional rules for making the social choice in any given situation, e.g., a religious code.

The last two methods of social choice, dictatorship and convention, have in their formal structure a certain definiteness absent from voting or the market mechanism. In an ideal dictatorship, there is but one will involved in choice; in an ideal society ruled by convention, there is but the divine will or perhaps, by assumption, a common will of all individuals concerning social decisions, so that in either case no conflict of individual wills is involved. The methods of voting and of the market, on the other hand, are methods of amalgamating the tastes of many individuals in the making of social choices. The methods of dictatorship and convention are, or can be, rational in the sense that any individual can be rational in his choice. Can such consistency be attributed to collective modes of choice, where the wills of many people are involved?

It should be emphasized here that the present study is concerned only with the formal aspects of the foregoing question. That is, we ask if it is formally possible to construct a procedure for passing from a set of known individual tastes to a pattern of social decision-making, the procedure in question being required to satisfy certain natural conditions. An illustration of the problem is the following well-known "paradox of voting." Suppose there is a community consisting of three voters

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and this community must choose among three alternative modes of social action (e.g., disarmament, cold war, or hot war). It is expected that choices of this type have to be made repeatedly, but sometimes not all of the three alternatives will be available. In analogy with the usual utility analysis of the individual consumer under conditions of constant wants and variable price-income situations, rational behavior on the part of the community would mean that the community orders the three alternatives according to its collective preferences once for all and then chooses in any given case that alternative among those actually available which stands highest on this list. A natural way of arriving at the collective preference scale would be to say that one alternative is preferred to another if a majority of the community prefer the first alternative to the second, i.e., would choose the first over the second if those were the only two alternatives. Let A, B, and C be the three alternatives, and 1, 2, and 3 the three individuals. Suppose individual 1 prefers A to B and B to C (and therefore A to C), individual 2 prefers B to C and C to A (and therefore B to A), and individual 3 prefers C to A and A to B (and therefore C to B). Then a majority prefers A to B, and a majority prefers B to C. We may therefore say that the community prefers A to B and B to C. If the community is to be regarded as behaving rationally, we are forced to say that A is preferred to C. But, in fact, a majority of the community prefers C to A.[2] So the method just outlined for passing from individual to collective tastes fails to satisfy the condition of rationality as we ordinarily understand it. Can we find other methods of aggregating individual tastes which imply rational behavior on the part of the community and which will be satisfactory in other ways?[3]

If we adopt the traditional identification of rationality with maximization of some sort, then the problem of achieving a social maximum derived from individual desires is precisely the problem which has been central to the field of welfare economics.[4] However, the search for a clear definition of optimum social welfare has been plagued by the difficulties of interpersonal comparisons. The emphasis, as is well known, has shifted to a weaker definition of optimum, namely, the determination of all social states such that no individual can be made better off without making someone worse off. As Professors Bergson, Lange, and Samuelson have argued, though, the weaker definition cannot be used as a guide to social policy; the second type of welfare economics is only important as a prelimi-

[2] It may be added that the method of decision sketched above is essentially that used in deliberative bodies, where a whole range of alternatives usually comes up for decision in the form of successive pairwise comparisons. The phenomenon described in the text can be seen in a pure form in the disposition of the proposals before recent Congresses for federal aid to state education, the three alternatives being no federal aid, federal aid to public schools only, and federal aid to both public and parochial schools.

[3] The problem of collective rationality has been discussed by Professor Frank H. Knight, but chiefly in terms of the socio-psychological prerequisites; see "The Planful Act: The Possibilities and Limitations of Collective Rationality," in *Freedom and Reform* (New York: Harper & Bros., 1947), pp. 335–69, esp. pp. 346–65).

[4] See P. A. Samuelson, *Foundations of Economic Analysis* (Cambridge, Mass.: Harvard University Press, 1947), chap. viii; A. Bergson (Burk), "A Reformulation of Certain Aspects of Welfare Economics," *Quarterly Journal of Economics,* LII (1938), 310–34; O. Lange, "The Foundations of Welfare Economics," *Econometrica,* X (1942), 215–28; M. W. Reder, *Studies in the Theory of Welfare Economics* (New York, 1947), chaps. i–v.

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nary to the determination of a genuine social maximum in the full sense. E.g., under the usual assumptions, if there is an excise tax imposed on one commodity in the initial situation, it can be argued that the removal of the tax accompanied by a suitable redistribution of income and direct tax burdens will improve the position of all individuals in the society. But there are, in general, many redistributions which will accomplish this end, and society must have some criterion for choosing among them before it can make any change at all. Further, there is no reason for confining the range of possible social actions to those which will injure no one as compared with the initial situation, unless the status quo is to be sanctified on ethical grounds. All we can really say is that society ought to abolish the excise tax and make some redistribution of income and tax burdens; but this is no prescription for action unless there is some principle by which society can make its choice among attainable income distributions, i.e., a social indifference map.

Voting can be regarded as a method of arriving at social choices derived from the preferences of individuals. Another such method of more specifically economic content is the compensation principle, as proposed by Mr. Kaldor:[5] in a choice between two alternative economic states x and y, if there is a method of paying compensations under state x such that everybody can be made better off in the state resulting from making the compensations under x than they are in state y, then x should be chosen in preference to y, *even if the compensation is not actually paid.* Apart from the ethical difficulties in the acceptance of this principle,[6] there is a formal difficulty which was pointed out by Professor Scitovszky:[7] it is possible that simultaneously x should be preferred to y and y be preferred to x. Just as in the case of majority voting, this method of aggregating individual preferences may lead to a pattern of social choice which is not a linear ordering of the social alternatives. Note that in both cases the paradox need not occur; all that is said is that there are preference patterns which, if held by the individual members of the society, will give rise to an inconsistent pattern of social choice. Unless the trouble-breeding individual preference patterns can be ruled out by a priori assumption, both majority voting and the compensation principle must be regarded as unsatisfactory techniques for the determination of social preferences.

The aim of the present paper is to show that these difficulties are general. For *any* method of deriving social choices by aggregating individual preference patterns which satisfies certain natural conditions, it is possible to find individual preference patterns which give rise to a social choice pattern which is not a linear ordering. In particular, this is very likely to be the case if, as is frequently assumed, each individual's preferences among social states are derived purely from his personal consumption-leisure-saving situation in each.[8] It is assumed that individuals act rationally, in the sense that their be-

[5] N. Kaldor, "Welfare Propositions of Economics and Interpersonal Comparisons of Utility," *Economic Journal,* XLIX (1939), 549–52; see also J. R. Hicks, "The Foundations of Welfare Economics," *Economic Journal,* XLIX (1939), 698–701 and 711–12.

[6] See W. J. Baumol, "Community Indifference," *Review of Economic Studies,* XIV (1946–47), 44–48.

[7] T. Scitovszky, "A Note on Welfare Propositions in Economics," *Review of Economic Studies,* IX (1942), 77–88.

[8] See, e.g., Samuelson, *op. cit.,* pp. 222–24; Bergson, *op. cit.,* pp. 318–20; Lange, *op. cit.,* p. 216.

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havior in alternative situations can be described by an indifference map. It is further assumed that utility is not measurable in any sense relevant to welfare economics, so that the tastes of an individual are completely described by a suitable preference pattern or indifference map.

II. DEFINITIONS AND NOTATION

1. A NOTATION FOR PREFERENCES AND CHOICE

In this paper I shall be interested in the description of preference patterns both for the individual and for society. It will be found convenient to represent preference by a notation not customarily employed in economics, though familiar in mathematics and particularly in symbolic logic. We assume that there is a basic set of alternatives which could conceivably be presented to the chooser. In the theory of consumers' choice, each alternative would be a commodity bundle; in the theory of the firm, each alternative would be a complete decision on all inputs and outputs; in welfare economics, each alternative would be a distribution of commodities and labor requirements. These alternatives are mutually exclusive; they are denoted by small letters, $x, y, z, \ldots$. On any given occasion the chooser has available to him a subset S of all possible alternatives, and he is required to choose one out of this set. The set S is a generalization of the well-known opportunity curve; thus, in the theory of consumer's choice under perfect competition, it would be the budget plane. It is assumed further that the choice is made in this way: Before knowing the set S, the chooser considers in turn all possible pairs of alternatives, say x and y, and for each pair he makes one and only one of three decisions: x is preferred to y, x is indifferent to y, or y is preferred to x. The decisions made for different pairs are assumed to be consistent with one another, so that, for example, if x is preferred to y and y to z, then x is preferred to z; similarly, if x is indifferent to y and y to z, then x is indifferent to z. Having this ordering of all possible alternatives, the chooser is now confronted with a particular opportunity set S. If there is one alternative in S which is preferred to all others in S, then the chooser selects that one alternative.[9]

Preference and indifference are relations between alternatives. Instead of working with two relations, it will be slightly more convenient to use a single relation, "preferred or indifferent." The statement, "x is preferred or indifferent to y," will be symbolized by xRy. The letter R, by itself, will be the name of the relation and will stand for a knowledge of all pairs such that xRy. From our previous discussion, we have, for any pair of alternatives x and y, either that x is preferred to y or y to x or that the two are indifferent. That is, we have assumed that any two alternatives are comparable. But this assumption may be written symbolically,

Axiom I: For all x and y, either xRy or yRx.

Note that Axiom I is presumed to hold when $x = y$, as well as when x is distinct from y, for we ordinarily say that x is indifferent to itself for any x, and this implies xRx. Note also that the

[9] It may be that there is a subset of alternatives in S, such that the alternatives in the subset are each preferred to every alternative not in the subset, while the alternatives in the subset are indifferent to one another. This case would be one in which the highest indifference curve which has a point in common with a given opportunity curve has at least two points in common with it (the well-known case of multiple maxima). In this case, the best thing to say is that the choice made in S is the whole subset; the first case discussed is one in which the subset in question, the choice, contains a single element.

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word "or" in the statement of Axiom I does not exclude the possibility of both xRy and yRx. That word merely asserts that at least one of the two events must occur; both may.

The property mentioned above of consistency in the preferences as between different pairs of alternatives may be stated more precisely, as follows: if x is preferred or indifferent to y and y is preferred or indifferent to z, then x must be either preferred or indifferent to z. In symbols,

Axiom II: For all x, y, and z, xRy and yRz imply xRz.

A relation satisfying both Axiom I and Axiom II is termed a weak ordering or sometimes simply an ordering. It is clear that a relation having these two properties taken together does create a ranking of the various alternatives. The adjective "weak" refers to the fact that the ordering does not exclude indifference, i.e., Axioms I and II do not exclude the possibility that for some distinct x and y, both xRy and yRx.

It might be held that the two axioms in question do not completely characterize the concept of a preference pattern. For example, we ordinarily feel that not only the relation R but also the relations of (strict) preference and of indifference satisfy Axiom II. It can be shown that, by defining preference and indifference suitably in terms of R, it will follow that all the usually desired properties of preference patterns obtain.

Definition 1: xPy is defined to mean not yRx.

The statement "xPy" is read, "x is preferred to y."

Definition 2: xIy means xRy and yRx.

The statement "xIy" is read, "x is indifferent to y." It is clear that P and I, so defined, correspond to the ordinary notions of preference and indifference, respectively.

Lemma: *a*) For all x, xRx.
b) If xPy, then xRy.
c) If xPy and yPz, then xPz.
d) If xIy and yIz, then xIz.
e) For all x and y, either xRy or yPx.
f) If xPy and yRz, then xPz.

All these statements are intuitively self-evident from the interpretations placed on the symbols.

For clarity, we shall avoid the use of the terms "preference scale" or "preference pattern" when referring to R, since we wish to avoid confusion with the concept of preference proper, denoted by P. We shall refer to R as an "ordering relation" or "weak ordering relation" or, more simply, as an "ordering" or "weak ordering." The term "preference relation" will refer to the relation P.

Suppose that we know the choice which would be made from any given pair of alternatives; i.e., given two alternatives x and y from which the chooser must select, we know whether he would take x or y or remain indifferent between them. Since choosing x from the pair x, y implies that x is preferred to y, and similarly with a choice of y, a knowledge of the choice which would be made from any two given alternatives implies a knowledge of the full preference scale; from earlier remarks this, in turn, implies a knowledge of the choice which would be made from any set of alternatives actually available. Hence, one of the consequences of the assumption of rational behavior is that the choice from any collection of alternatives can be determined by a knowledge of the choices

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which would be made from pairs of alternatives.

2. THE ORDERING OF SOCIAL STATES

In the present study the objects of choice are social states. The most precise definition of a social state would be a complete description of the amount of each type of commodity in the hands of each individual, the amount of labor to be applied by each individual, the amount of each productive resource invested in each type of productive activity, and the amounts of various types of collective activity such as municipal services, diplomacy and its continuation by other means, and the erection of statues to famous men. It is assumed that each individual in the community has a definite ordering of all conceivable social states in terms of their desirability to him. It need not be assumed here that an individual's attitude toward different social states is determined exclusively by the commodity bundles which accrue to his lot under each. The individual may order all social states by whatever standards he deems relevant. A member of Veblen's leisure class might order the states solely on the criterion of his relative income standing in each; a believer in the equality of man might order them in accordance with some measure of income equality. Indeed, since, as mentioned above, some of the components of the social state, considered as a vector, are collective activities, purely individualistic assumptions are useless in analyzing such problems as the division of the national income between public and private expenditure. The present notation permits perfect generality in this respect. Needless to say, this generality is not without its price. More information would be available for analysis if the generality were restricted by a prior knowledge of the nature of individual orderings of social states. This problem will be touched on again.

In general, then, there will be a difference between the ordering of social states according to the direct consumption of the individual and the ordering when the individual adds his general standards of equity (or perhaps his standards of pecuniary emulation).[10] We may refer to the former ordering as reflecting the *tastes* of the individual and the latter as reflecting his *values*. The distinction between the two is by no means clear cut. An individual with aesthetic feelings certainly derives pleasure from his neighbor's having a well-tended lawn. Under the system of a free market, such feelings play no direct part in social choice; yet, psychologically, they differ only slightly from the pleasure in one's own lawn. Intuitively, of course, we feel that not all the possible preferences which an individual might have ought to count; his preferences for matters which are "none of his business" should be irrelevant. Without challenging this view, I should like to emphasize that the decision as to which preferences are relevant and which are not is itself a value judgment and cannot be settled on an a priori basis. From a formal point of view, one cannot distinguish between an individual's dislike of having his grounds ruined by factory smoke and his extreme distaste for the existence of heathenism in Central Africa. There are probably not a few individuals in this country who would regard the former feeling as irrelevant for social policy and the latter as relevant, though

[10] This distinction has been stressed to the author by M. Friedman, University of Chicago.

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the majority would probably reverse the judgment. I merely wish to emphasize here that we must look at the entire system of values, including values about values, in seeking for a truly general theory of social welfare.

It is the ordering according to values which takes into account all the desires of the individual, including the highly important socializing desires, and which is primarily relevant for the achievement of a social maximum. The market mechanism, however, takes into account only the ordering according to tastes. This distinction is the analogue, on the side of consumption, of the divergence between social and private costs in production which has been developed by Professor Pigou.[11]

As for notation, let R_i be the ordering relation for alternative social states from the standpoint of individual i. Sometimes, when several different ordering relations are being considered for the same individual, the symbols will be distinguished by adding a superscript. Corresponding to the ordering relation R_i, we have the (strict) preference relation P_i and the indifference relation I_i. If the symbol for the ordering has a prime or second attached (thus, R_i', R_i''), then the corresponding symbols for preference and indifference will have the prime or second attached, respectively.

Similarly, society as a whole will be considered provisionally to have a social ordering relation for alternative social states, which will be designated by R, sometimes with a prime or second. Social preference and indifference will be denoted by P and I, respectively, primes or seconds being attached when they are attached to the relation R, respectively.

Throughout this analysis, it will be assumed that individuals are rational, by which is meant that the ordering relations R_i satisfy Axioms I and II. The problem will be to construct an ordering relation for society as a whole which is also to reflect rational choice-making, so that R also will be assumed to satisfy Axioms I and II.

III. THE SOCIAL WELFARE FUNCTION

1. FORMAL STATEMENT OF THE PROBLEM OF SOCIAL CHOICE

I shall largely restate Bergson's formulation of the problem of making welfare judgments[12] in the terminology here adopted. The various arguments of his social welfare function are the components of what I have here termed the "social state," so that essentially he is describing the process of assigning a numerical social utility to each social state, the aim of society then being described by saying it seeks to maximize the social utility or social welfare subject to whatever technological or resource constraints are relevant, or, put otherwise, it chooses the social state yielding the highest possible social welfare within the environment. As with any type of behavior described by maximization, the measurability of social welfare need not be assumed; all that matters is the existence of a social ordering satisfying Axioms I and II. As before, all that is needed to define such an ordering is to know the relative ranking of each pair of alternatives.

The relative ranking of a fixed pair

[11] A. C. Pigou, *The Economics of Welfare* (London: Macmillan & Co., 1920), Part II, chap. vi. For the analogy see Samuelson, *op. cit.*, p. 224; Reder, *op. cit.*, pp. 64–67; G. Tintner, "A Note on Welfare Economics," *Econometrica*, XIV (1946), 69–78.

[12] Bergson, *op. cit.*

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of alternative social states will vary, in general, with changes in the values of at least some individuals; to assume that the ranking does not change with any changes in individual values is to assume, with traditional social philosophy of the Platonic realist variety, that there exists an objective social good defined independently of individual desires. This social good, it was frequently held, could be best apprehended by the methods of philosophic inquiry. Such a philosophy could be and was used to justify government by elite, secular or religious, although the connection is not a necessary one.

To the nominalist temperament of the modern period the assumption of the existence of the social ideal in some Platonic realm of being was meaningless. The utilitarian philosophy of Jeremy Bentham and his followers sought instead to ground the social good on the good of individuals. The hedonist psychology associated with utilitarian philosophy was further used to imply that each individual's good was identical with his desires. Hence, the social good was in some sense to be a composite of the desires of individuals. A viewpoint of this type serves as a justification of both political democracy and laissez faire economics or at least an economic system involving free choice of goods by consumers and of occupations by workers.

The hedonist psychology finds its expression here in the assumption that individuals' behavior is expressed by individual ordering relations R_i. Utilitarian philosophy is expressed by saying for each pair of social states that the choice depends on the ordering relations of all individuals, i.e., depends on $R_1, \ldots, R_n$, where n is the number of individuals in the community. Put otherwise, the whole social ordering relation R is to be determined by the individual ordering relations for social states, $R_1, \ldots, R_n$. We do not exclude here the possibility that some or all of the choices between pairs of social states made by society might be independent of the preferences of certain particular individuals, just as a function of several variables might be independent of some of them.

Definition 3: By a "social welfare function" will be meant a process or rule which, for each set of individual orderings $R_1, \ldots, R_n$ for alternative social states (one ordering for each individual), states a corresponding social ordering of alternative social states, R.

As a matter of notation, we shall let R be the social ordering corresponding to the set of individual orderings $R_1, \ldots, R_n$, the correspondence being that established by a given social welfare function; if primes or seconds are added to the symbols for the individual orderings, primes or seconds will be added to the symbol for the corresponding social ordering.

There is some difference between the concept of social welfare function used here and that employed by Bergson. The individual orderings which enter as arguments into the social welfare function as defined here refer to the values of individuals rather than to their tastes. Bergson supposes individual values to be such as to yield a social value judgment leading to a particular rule for determining the allocation of productive resources and the distribution of leisure and final products in accordance with individual tastes. In effect, the social welfare function described here is a method of choosing which social welfare function of the Bergson type will be applicable, though of course I do not ex-

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clude the possibility that the social choice actually arrived at will not be consistent with the particular value judgments formulated by Bergson. But in the formal aspect the difference between the two definitions of social welfare function is not too important. In Bergson's treatment the tastes of individuals (each for his own consumption) are represented by utility functions, i.e., essentially by ordering relations; hence, the Bergson social welfare function is also a rule for assigning to each set of individual orderings a social ordering of social states. Further, as already indicated, no sharp line can be drawn between tastes and values.

A special type of social welfare function would be one which assigns the same social ordering for every set of individual orderings. In this case, of course, social choices are completely independent of individual tastes, and we are back in the Platonic case.

For simplicity of exposition, it will be assumed that the society under study contains only two individuals and that the total number of alternatives which are conceivable is three. Since the results to be obtained are negative, the latter restriction is not a real one; if it turns out to be impossible to construct a social welfare function which will define a social ordering of three alternatives, it will a fortiori be impossible to define one which will order more alternatives. The restriction to two individuals may be more serious; it is conceivable that there may be suitable social welfare functions which can be defined for three individuals but not for two, for example. In fact, this is not so, and the results stated in this paper hold for any number of individuals. However, the proof will be considerably simplified by considering only two.

We shall not ask, in general, that the social welfare function be defined for every logically possible set of individual orderings. On a priori grounds we may suppose it known that preferences for alternative social states are formed only in a limited set of ways, and the social welfare function need only be defined for individual orderings formed in those ways. For example, we may suppose (and will later on) that each individual orders social alternatives according to his own personal consumption under each (the purely individualistic case). Then the social welfare function need be defined only for those sets of individual orderings which are admissible, in the sense of being consistent with our a priori assumptions about the empirical possibilities.

Condition 1: The social welfare function is defined for every admissible pair of individual orderings, R_1, R_2.

Condition 1, it should be emphasized, is a restriction on the form of the social welfare function, since we are requiring that for some sufficiently wide range of sets of individual orderings, the social welfare function give rise to a true social ordering.

2. POSITIVE ASSOCIATION OF SOCIAL AND INDIVIDUAL VALUES

Since we are trying to describe social "welfare" and not some sort of "illfare," we must assume that the social welfare function is such that the social ordering responds positively to alterations in individual values or at least not negatively. Hence, we may state the following condition:

Condition 2: If an alternative social state x rises or does not fall in the ordering of each

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individual without any other change in those orderings and if x was preferred to another alternative y before the change in individual orderings, then x is still preferred to y.

3. THE INDEPENDENCE OF IRRELEVANT ALTERNATIVES

Just as for a single individual, the choice made by society from any given set of alternatives should be independent of the very existence of alternatives outside the given set. For example, suppose an election system has been devised whereby each individual lists all the candidates in order of his preference, and then, by a preassigned procedure, the winning candidate is derived from these lists. (All actual election procedures are of this type, although in most the entire list is not required for the choice.) Suppose an election is held, with a certain number of candidates in the field, each individual filing his list of preferences, and then one of the candidates dies. Surely, the social choice should be made by taking each of the individual's preference lists, blotting out completely the dead candidate's name, and considering only the orderings of the remaining names in going through the procedure of determining the winner. That is, the choice to be made among the set of surviving candidates should be independent of the preferences of individuals for the nonsurviving candidates. To assume otherwise would be to make the result of the election dependent on the obviously accidental circumstance of whether a candidate died before or after the date of polling. Therefore, we may require of our social welfare function that the choice made by society from a given set of alternatives depend only on the orderings of individuals among those alternatives. Alternatively stated, if we consider two sets of individual orderings such that, for each individual, his ordering of those particular alternatives under consideration is the same each time, then we require that the choice made by society be the same if individual values are given by the first set of orderings as if they are given by the second.

Condition 3: Let R_1, R_2, and R_1', R_2' be two sets of individual orderings. If, for both individuals i and for all x and y in a given set of alternatives S, xR_iy if and only if $xR_i'y$, then the social choice made from S is the same whether the individual orderings are R_1, R_2, or R_1', R_2'. (Independence of irrelevant alternatives.)

The reasonableness of this condition can be seen by consideration of the possible results in a method of choice which does not satisfy Condition 3, the rank-order method of voting frequently used in clubs.[13] With a finite number of candidates, let each individual rank all his candidates, i.e., designate his first-choice candidate, second-choice candidate, etc. Let preassigned weights be given first, second, etc., choices, the higher weight to the higher choice, and then let the candidate with the highest weighted sum of votes be elected. In particular, suppose there are three voters and four candidates, x, y, z, and w. Let the weights for first, second, third, and fourth choices be 4, 3, 2, and 1, respectively. Suppose that individuals 1 and 2 rank the candidates in the order x, y, z, and w, while individual 3 ranks them in the order z, w, x, and y. Under the given electoral system, x is chosen. Then, certainly, if y is deleted from the ranks of the candidates, the system applied to the remaining candidates should yield the same result,

[13] This example was suggested by a discussion with G. E. Forsythe, National Bureau of Standards.

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especially since, in this case, y is inferior to x according to the tastes of every individual; but, if y is in fact deleted, the indicated electoral system would yield a tie between x and z.

The condition of the independence of irrelevant alternatives implies that in a generalized sense all methods of social choice are of the type of voting. If S is the set consisting of the two alternatives x and y, Condition 3 tells us that the choice between x and y is determined solely by the preferences of the members of the community as between x and y. That is, if we know which members of the community prefer x to y, which are indifferent, and which prefer y to x, then we know what choice the community makes. Knowing the social choices made in pairwise comparisons in turn determines the entire social ordering and therewith the social choice made from any set of alternatives. Condition 2 guarantees that voting for a certain alternative has the usual effect of making surer that that alternative will be adopted.

Condition 1 says, in effect, that, as the set of alternatives varies and individual orderings remain fixed, the different choices made shall bear a certain type of consistent relation to one another. Conditions 2 and 3, on the other hand, suppose a fixed set of alternatives and say that for certain particular types of variation in individual values, the various choices made have a certain type of consistency.

4. THE CONDITION OF CITIZENS' SOVEREIGNTY

We certainly wish to assume that the individuals in our society be free to choose, by varying their values, among the alternatives available. That is, we do not wish our social welfare function to be such as to prevent us, by its very definition, from expressing a preference for some given alternative over another.

Definition 4: A social welfare function will be said to be *imposed* if for some pair of distinct alternatives x and y, xRy for any set of individual orderings R_1, R_2, where R is the social ordering corresponding to R_1, R_2.

In other words, when the social welfare function is imposed, there is some pair of alternatives x and y such that the community can never express a preference for y over x no matter what the tastes of both individuals are, indeed even if both individuals prefer y to x; some preferences are taboo. (Note that, by Definition 1, asserting that xRy holds for all sets of individual orderings is equivalent to asserting that yPx never holds.) We certainly wish to require of our social welfare function the condition that it not be imposed in the sense of Definition 4; we certainly wish all choices to be possible if unanimously desired by the group.

Condition 4: The social welfare function is not to be imposed.

Condition 4 is stronger than need be for the present argument. Some decisions, as between given pairs of alternatives, may be assumed to be imposed. All that is required really is that there be a set S of three alternatives such that the choice between any pair is not constrained in advance by the social welfare function.

It should also be noted that Condition 4 excludes the Platonic case discussed in section 1 of Part III above. It expresses fully the idea that all social choices are determined by individual desires. In conjunction with Condition 2 (which insures that the determination is in the direction of agreeing with individual desires), Condition 4

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expresses the same idea as Professor Bergson's Fundamental Value Propositions of Individual Preference, which state that of two alternatives between which all individuals but one are indifferent, the community will prefer one over the other or be indifferent between the two according as the one individual prefers one over the other or is indifferent between the two.[14] Conditions 2 and 4 together correspond to the usual concept of consumers' sovereignty; since we are here referring to values rather than to tastes, we might refer to them as expressing the idea of citizens' sovereignty.

5. THE CONDITION OF NONDICTATORSHIP

A second form of social choice not of a collective character is the choice by dictatorship. In its pure form this means that social choices are to be based solely on the preferences of one man. That is, whenever the dictator prefers x to y, so does society. If the dictator is indifferent between x and y, presumably he will then leave the choice up to some or all of the other members of society.

Definition 5: A social welfare function is said to be "dictatorial" if there exists an individual i such that for all x and y, xP_iy implies xPy regardless of the orderings of all individuals other than i, where P is the social preference relation corresponding to those orderings.

Since we are interested in the construction of collective methods of social choice, we wish to exclude dictatorial social welfare functions.

[14] Bergson, *op. cit.*, pp. 318–20. The Fundamental Value Propositions of Individual Preference are not, strictly speaking, implied by Conditions 2 and 4 (in conjunction with Conditions 1 and 2), although something very similar to them is so implied; see Consequence 1 in Part IV, section 2 below. A slightly stronger form of Condition 2 than that stated here would suffice to yield the desired implication.

Condition 5: The social welfare function is not to be dictatorial (nondictatorship).

We have now imposed five apparently reasonable conditions on the construction of a social welfare function. These conditions are of course value judgments and could be called into question; taken together, they express the doctrines of citizens' sovereignty and rationality in a very general form, with the citizens being allowed to have a wide range of values. The question is that of constructing a social ordering of all conceivable alternative social states from any given set of individual orderings of those social states, the method of construction being in accordance with the value judgments of citizens' sovereignty and rationality as expressed in Conditions 1–5.

IV. THE POSSIBILITY THEOREM FOR SOCIAL WELFARE FUNCTIONS

1. THE RANGE OF POSSIBLE INDIVIDUAL ORDERINGS

For simplicity we shall impose on the individual preference scales two conditions which in fact have almost invariably been assumed in works on welfare economics: (1) each individual's comparison of two alternative social states depends only on the commodities that he receives (and labor that he expends) in the two states, i.e., he is indifferent as between any two social states in which his own consumption-leisure-saving situations are the same or at least indifferent to him; (2) in comparing two personal situations in one of which he receives at least as much of each commodity (including leisure and saving as commodities) and more of at least one commodity than in the other, the individual will prefer the first situation. Suppose that among the possible alternatives

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there were three, none of which gave any individual at least as much of both commodities as any other. For example, suppose that there are two individuals and a total of ten units of each of two commodities. Consider three alternative distributions described by the accompanying tabulation. The individualistic

Alternative	Individual 1		Individual 2	
	Commodity 1	Commodity 2	Commodity 1	Commodity 2
1	5	1	5	9
2	4	2	6	8
3	3	3	7	7

restrictions imposed do not tell us anything about the way either individual orders these alternatives. Under the individualistic assumptions there is no a priori reason to suppose that the two individuals will not order the alternatives in any given way. In the sense of Part III, section 1, above, all individual orderings of the three alternatives are admissible. Condition 1 therefore requires that the social welfare function be defined for all pairs of individual orderings, R_1, R_2.

2. THE POSSIBILITY THEOREM

Some consequences will be drawn from Conditions 1–5 for the present case of a social welfare function for two individuals and three alternatives. It will be shown that the supposition that there is a social welfare function satisfying those conditions leads to a contradiction.

Let x, y, and z be the three alternatives among which choice is to be made, e.g., three possible distributions of commodities. Let x' and y' be variable symbols which represent possible alternatives, i.e., range over the values x, y, z. Let the individuals be designated as 1 and 2, and let R_1 and R_2 be the orderings by 1 and 2, respectively, of the alternatives x, y, z. Let P_1 and P_2 be the corresponding preference relations; e.g., $x'P_1y'$ means that individual 1 strictly prefers x' to y'.

Consequence 1: If $x'P_1y'$ and $x'P_2y'$, then $x'Py'$.

I.e., if both prefer x' to y', then society must prefer x' to y'.

Proof.—By Condition 4 there are orderings R_1' and R_2', for individuals 1 and 2, respectively, such that, in the corresponding social preference, $x'P'y'$. Form R_1'' from R_1' by raising x', if need be, to the top, while leaving the relative positions of the other two alternatives alone; form R_2'' from R_2' in the same way. Since all we have done is raise alternative x' in everyone's esteem, while leaving the others alone, x' should still be preferred to y' by society in accordance with Condition 2, so that $x'P''y'$. But, by construction, both individuals prefer x' to y' in the orderings R_1'', R_2'', and society prefers x' to y'. Since, by Condition 3, the social choice between x' and y' depends only on the individual orderings of those two alternatives, it follows that whenever both individuals prefer x' to y', regardless of the rank of the third alternative, society will prefer x' to y', which is the statement to be proved.

Consequence 2: Suppose that for some x' and y', whenever $x'P_1y'$ and $y'P_2x'$, $x'Py'$. Then, for that x' and y', whenever $x'P_1y'$, $x'Py'$.

I.e., if in a given choice, the will of individual 1 prevails against the opposition of 2, then individual 1's views will certainly prevail if 2 is indifferent or if he agrees with 1.

Proof.—Let R_1 be an ordering in which $x'P_1y'$, R_2 be any ordering. Let

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R_1' be the same ordering as R_1, while R_2' is derived from R_2 by depressing x' to the bottom while leaving the relative positions of the other two alternatives unchanged. By construction, $x'P_1'y'$, $y'P_2'x'$. By hypothesis, $x'P'y'$, where P' is the social preference relation derived from the individual orderings R_1', R_2'. Now the only difference between R_1', R_2' and R_1, R_2 is that x' is raised in the scale of individual 2 in the latter as compared with the former. Hence, by Condition 2 (interchanging the R_i's and the R_i''s) it follows from $x'P'y'$ that $x'Py'$. I.e., whenever R_1, R_2 are such that $x'P_1y'$, then $x'Py'$.

Consequence 3: If $x'P_1y'$ and $y'P_2x'$, then $x'Iy'$.

I.e., if the two individuals have exactly opposing interests on the choice between two given alternatives, then society will be indifferent between the alternatives.

PROOF.—Suppose the consequence is false. Then, for some orderings R_1 and R_2 and for some pair of alternatives x' and y', we would have $x'P_1y'$, $y'P_2x'$, but not $x'Iy'$. In that case, either $x'Py'$ or $y'Px'$. We will suppose $x'Py'$ and show that this supposition leads to a contradiction; the same reasoning would show that the assumption $y'Px'$ also leads to a contradiction.

Without loss of generality it can be assumed that x' is the alternative x, $y' = y$. Then we have, for the particular orderings in question, xP_1y, yP_2x, and xPy. Since the social choice between x and y depends, by Condition 3, only on the individual choices as between x and y, we must have

$$\text{whenever } xP_1y \text{ and } yP_2x,\ xPy\ . \qquad (1)$$

It will be shown that (1) leads to a contradiction.

Suppose individual 1 prefers x to y and y to z, while individual 2 prefers y to z and z to x. Individual 2 then prefers y to x. By (1) society prefers x to y. Also, both prefer y to z; by Consequence 1, society prefers y to z. Since society prefers x to y and y to z, it must prefer x to z. Therefore, we have exhibited orderings R_1, R_2 such that xP_1z, zP_2x, but xPz. Since the social choice between x and z depends only on the individual preferences for x and z,

$$\text{whenever } xP_1z \text{ and } zP_2x,\ xPz\ . \qquad (2)$$

Now suppose R_1 is the ordering y, x, z, and R_2 the ordering z, y, x. By Consequence 1, yPx; by (2) xPz, so that yPz. By the same reasoning as before,

$$\text{whenever } yP_1z \text{ and } zP_2y,\ yPz\ . \qquad (3)$$

If R_1 is the ordering y, z, x, and R_2 the ordering z, x, y, it follows from Consequence 1 and (3) that zPx and yPz, so that yPx. Hence,

$$\text{whenever } yP_1x \text{ and } xP_2y,\ yPx\ . \qquad (4)$$

If R_1 is the ordering z, y, x, and R_2 the ordering x, z, y, then from Consequence 1 and (4), zPy and yPx, so that zPx.

$$\text{Whenever } zP_1x \text{ and } xP_2z,\ zPx\ . \qquad (5)$$

If R_1 is the ordering z, x, y, and R_2 x, y, z, then, using (5), zPx and xPy, so that zPy.

$$\text{Whenever } zP_1y \text{ and } yP_2z,\ zPy\ . \qquad (6)$$

From (1) it follows from Consequence 2 that whenever xP_1y, xPy. Similarly, from (1) to (6) it follows that for any pair of alternatives x', y', whenever $x'P_1y'$, then $x'Py'$. That is, by Definition 5, individual 1 would be a dictator. This is prohibited by Condition 5, so that (1) must be false. Therefore, Consequence 3 is proved.

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Now suppose individual 1 has the ordering x, y, z, while individual 2 has the ordering z, x, y. By Consequence 1,

$$xPy\,. \tag{7}$$

Since yP_1z, zP_2y, it follows from Consequence 3 that

$$yIz\,. \tag{8}$$

From (7) and (8), xPz. But, also xP_1z, zP_2x, which implies xIz by Consequence 3. It cannot be that x is both preferred and indifferent to z. Hence the assumption that there is a social welfare function compatible with Conditions 1–5 has led to a contradiction.

Put another way, if we assume that our social welfare function satisfies Conditions 2–3 and we further suppose that Condition 1 holds, then either Condition 4 or Condition 5 must be violated. Condition 4 states that the social welfare function is not imposed; Condition 5 states that it is not dictatorial.

Possibility Theorem.—If there are at least three alternatives among which the members of the society are free to order in any way, then every social welfare function satisfying Conditions 2 and 3 and yielding a social ordering satisfying Axioms I and II must be either imposed or dictatorial.[15] The Possibility Theorem shows that, if no prior assumptions are made about the nature of individual orderings, there is no method of voting which will remove the paradox of voting discussed in Part I, neither plurality voting nor any scheme of proportional representation, no matter how complicated. Similarly, the market mechanism does not create a rational social choice.

[15] The negative outcome expressed in this theorem is strongly reminiscent of the intransitivity of the concept of domination in the theory of multiperson games; see John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior* (2d ed.; Princeton University Press, 1947), pp. 38–39.

V. SOME IMPLICATIONS FOR THE FORMATION OF SOCIAL WELFARE JUDGMENTS

1. INTERPRETATION OF THE POSSIBILITY THEOREM

The interpretation of the Possibility Theorem is given by examination of the meaning of Conditions 1–5. In particular, it is required that the social ordering be formed from individual orderings and that the social decision between two alternatives be independent of the desires of individuals involving any alternatives other than the given two (Conditions 1 and 3). These conditions taken together serve to exclude interpersonal comparison of social utility either by some form of direct measurement or by comparison with other alternative social states. Therefore, the Possibility Theorem can be restated as follows:

If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial.

The word "satisfactory" in the foregoing statement means that the social welfare function does not reflect individuals' desires negatively (Condition 2) and that the resultant social tastes shall be represented by an ordering having the usual properties of rationality ascribed to individual orderings (Condition 1 and Axioms I and II).

In view of the interpretations placed on the conditions for a social welfare function in Part III above, we can also phrase the result this way: If con-

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sumers' values can be represented by a wide range of individual orderings, the doctrine of voters' sovereignty is incompatible with that of collective rationality.

If we wish to make social welfare judgments which depend on all individual values, i.e., are not imposed or dictatorial, then we must relax some of the conditions imposed. It will continue to be maintained that there is no meaningful interpersonal comparison of utilities and that the conditions wrapped up in the word "satisfactory" are to be accepted.[16] The only condition that remains to be eliminated is the one stating that the method of forming a social ordering would work properly for a wide range of sets of individual orderings. That is, it must be supposed that it is known in advance that the individual orderings $R_1, \ldots, R_n$ for social actions satisfy certain conditions more restrictive than those hitherto introduced.

2. A REFLECTION ON THE NEW WELFARE ECONOMICS

As noted in Part I, the so-called "new welfare economics" has concentrated on the determination of the totality of social states which have the property that any change which benefits one individual injures another—"maximal states" in Lange's terminology. In particular, this problem has usually been analyzed under the assumption that individual desires for social alternatives are formed in the individualistic way described above in Part IV, section 1. But if the only restrictions that we wish to impose on individual tastes are those implied by the individualistic assumptions, then, as we have seen, there is no satisfactory social welfare function possible when there is more than one commodity. Since, as we have seen, the only purpose of the determination of the maximal states is as a preliminary to the study of social welfare functions, the customary study of maximal states under individualistic assumptions is pointless. There is, however, a qualification which should be added. It is conceivable that, if further restrictions are added to the individualistic ones, a social welfare function will be possible. Any state which is maximal under the combination of individualistic and other restrictions will certainly be maximal if only individualistic restrictions are imposed on the individual orderings. Hence, if the proper handling of the social welfare problem is deemed to be the imposition of further restrictions in addition to the individualistic ones, then the social maximum in any given situation will be one of the maximal elements under the combined restrictions and hence one of the maximal elements under individualistic conditions. It is therefore not excluded that the current new welfare economics will be of some use in restricting the range in which we must look for the social maximum.

The failure of purely individualistic assumptions to lead to a well-defined social welfare function means, in effect, that there must be a divergence between social and private benefits if we are to be able to discuss a social optimum. Part of each individual's value system must be a scheme of socio-ethical norms, the realization of which cannot, by their nature, be achieved through atomistic market behavior. These norms, further, must be suffi-

[16] The only part of the last-named conditions that seems to me to be at all in dispute is the assumption of rationality. The consequences of dropping this assumption are so radical that it seems worth while to explore the consequences of maintaining it.

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ciently similar among the members of the society to avoid the difficulties outlined above.

3. A ONE-COMMODITY WORLD

The insufficiency of the individualistic hypotheses to permit the formation of a social welfare function, as developed in the previous sections, hinged on the assumption that there was more than one commodity involved. An investigation of the one-commodity case may be of interest to bring out more clearly the issues involved.

In a one-commodity world, if we make assumptions 1 and 2 of Part IV, section 1, there is for any given individual only one possible ordering of the social states. He orders various social states solely according to the amount of the one commodity he gets under each. In such a situation the individual orderings are not variables; Conditions 2, 3, and 4 become irrelevant, since they relate to the variation in the social ordering corresponding to certain specified types of changes in the individual orderings. Condition 5 (nondictatorship) becomes a much weaker restriction, though not completely irrelevant. Any specification of a social ordering which does not coincide completely with the ordering of any one individual will be a social welfare function compatible with all the conditions. For example, for each fixed total output, we might set up arbitrarily an ordering of the various distributions; then order any two social states with different total outputs in accordance with the total output, any two social states with the same total output according to the arbitrary ordering. This sets up a genuine weak ordering which does not coincide with the ordering of any one individual. For let x and y be two states with total outputs s and t, respectively, and apportionments s' and t', respectively, to the given individual. If $s > t$, but $s' < t'$, then society prefers x to y, while the individual prefers y to x.

The qualitative nature of the difference between the single- and multicommodity cases makes any welfare arguments based on an implicit assumption of a single commodity dubious in its applicability to real situations. The fundamental difficulty is that, in a world of more than one commodity, there is no unequivocal meaning to comparing total production in any two social states save in terms of some standard of value to make the different commodities commensurable; and, usually such a standard of value must depend on the distribution of income. In other words, there is no meaning to total output independent of distribution, i.e., of ethical judgments.

4. DISTRIBUTIONAL ETHICS COMBINED WITH INDIVIDUALISM

We may examine briefly a set of assumptions about individual values which seem to be common to those who feel that the new welfare economics is applicable in a fairly direct way to the solution of specific economic problems. It is assumed that there are (1) an accepted (let us say, unanimously accepted) value judgment that if everybody is better off (more precisely, if everybody is at least as well off and one person better off) in one social state than another *according to his tastes*, then the first social state is preferred to the second; and (2) a universally accepted ordering of different possible welfare distributions in any given situation. The latter value judgment usually takes an egalitarian form.

This ethical schema is quite explicit

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in the work of Bergson; the second value judgment is contained in his Propositions of Relative Shares.[17] The same set of ethics underlies the compensation principle of Professors Kaldor and Hicks. More recently, some proposals made by Professors Johnson and Modigliani for meeting the problem of the increased cost of food due to European demand seem to have been based on value judgments 1 and 2 above.[18] To prevent the inequitable shift in real income to farmers, it was proposed that there should be imposed an excise tax on food, accompanied by a per capita subsidy to consumers. Under the assumption that the supply of agricultural goods is completely inelastic, the tax would be absorbed by the farmers while the subsidy would have no substitution effects at the margin, so that the marginal rate of substitution for any pair of commodities would be the same for all consumers and hence the first value judgment would be fulfilled. The taxes and subsidies perform a purely distributive function and can be so arranged as to restore the status quo ante as near as may be, though actually the payment of a per capita subsidy implies a certain equalizing effect.

The value judgments are assumed here to hold for any individual. Note that even to state these judgments we must distinguish sharply between values and tastes (see Part II, sec. 2). All individuals are assumed to have the same values at any given instant of time, but the values held by any one individual will vary with variations in the tastes of all. Our previous arguments as to the nonexistence of social welfare functions were based on the diversity of values; do they carry over to this particular kind of unanimity?

The actual distribution of welfare dictated by the second value judgment cannot be stated simply in money terms. As Professor Samuelson points out, such a value judgment is not consistent with any well-defined social ordering of alternative social states.[19] The distribution of real income, for a given environment, must vary with individual tastes. Thus, for a given set of individual tastes (as represented by the ordering relations of all individuals, each for his own consumption) and a given environment, there is a given distribution of purchasing power (somehow defined); then exchange under perfectly competitive conditions proceeds until an optimum distribution is reached. The given distribution of real income and the individual tastes uniquely determine the final outcome, which is a social state. Therefore, the given ethical system is a rule which selects a social state as the choice from a given collection of alternative distributions of goods as a function of the tastes of all individuals. If, for a given set of tastes, the range of social alternatives varies, we expect that the choices will be consistent in the sense that the choice function is derivable from a social weak ordering of all social states. Thus, the ethical scheme discussed in this section, which we may term the "Bergson social welfare function," has the form of a rule assigning a social ordering to each possible set of individual orderings representing tastes. Mathematically, the

[17] Bergson, *op. cit.*

[18] D. G. Johnson, "The High Cost of Food—a Suggested Solution," *Journal of Political Economy*, LVI (1948), 54–57; Modigliani's proposals are contained in a press release of the Institute of World Affairs, New York, October, 1948.

[19] Samuelson, *op. cit.*, p. 225.

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Bergson social welfare function has, then, the same form as the social welfare function we have already discussed; though, of course, the interpretation is somewhat different, in that the individual orderings represent tastes rather than values and that the whole function is the end product of certain values assumed to be unanimously held rather than a method of reconciling divergent value systems. If the range of tastes is not restricted by a priori considerations (except that they must be truly tastes, i.e., refer only to an individual's own consumption, however that may be defined), then, indeed, the Bergson social welfare function is mathematically isomorphic to the social welfare function under individualistic assumptions. Hence the Possibility Theorem is applicable here; we cannot construct a Bergson social welfare function, i.e., cannot satisfy value judgments 1 and 2, which will satisfy Conditions 2–5 and which will yield a true social ordering for every set of individual tastes. Essentially, the two value judgments amount to erecting individualistic behavior into a value judgment. It is not surprising, then, that such ethics can be no more successful than the actual practice of individualism in permitting the formation of social welfare judgments.

It must of course be recognized that the meaning of Conditions 2–5 has changed. The previous arguments for their validity assumed that the individual orderings represented values rather than tastes. It seems obvious that Conditions 2, 4, and 5 have the same intrinsic desirability under either interpretation. Condition 3 is perhaps more doubtful. Suppose there are just two commodities, bread and wine. A distribution, deemed equitable by all, is arranged, with the wine-lovers getting more wine and less bread than the abstainers get. Suppose now that all the wine is destroyed. Are the wine-lovers entitled, because of that fact, to more than an equal share of bread? The answer is, of course, a value judgment. My own feeling is that tastes for unattainable alternatives should have nothing to do with the decision among the attainable ones; desires in conflict with reality are not entitled to consideration, so that Condition 3, reinterpreted in terms of tastes rather than of values, is a valid value judgment, to me at least.

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