Completed Coh. & p-adic LL:

§0: Motivation:

- A key feature of LL is its local-global compatibility. This is realized geometrically in the cohomology of Shimura vars., One form of this statement ~~is~~ for modular curves/forms is:

$$\cancel{A_{cusp}(K_f^p)_{\chi^{-1}} \cong \bigoplus_{\pi_\infty \otimes \pi_p} M(\pi_\infty \otimes \pi_p) \otimes \pi_\infty \otimes \pi_p,}$$

~~where: $\varinjlim_{K_p} A(K_p K_f^p)$; irred. adm. reps. ; Fixed A_∞° action.~~

Let $H^1_{par}(V_k) := \varinjlim_{K_f} H^1_{par}(Y(K_f)_{/\overline{\mathbb{Q}}}, V_k)$, over all cpct. open subgps of $GL_2(\mathbb{A}_f)$, $V_k := \mathrm{Sym}^{k-2} R^1 pr_* \mathbb{Z}_p$. (a ~~space of cusp. forms~~)

<u>Thm:</u> (~~E.~~ Carayol, Saito, see [Em-LGC, Thm. 2.5.1]) ?H-T?: Let $k \geq 2$. There is a $G_{\mathbb{Q}} \times GL_2(\mathbb{A}_f)$-equiv. isom.:

$$H^1_{par}(\overset{= \mathrm{Sym}^{k-2} R^1 pr_* \mathbb{Z}_p}{V_k})_{\overline{\mathbb{Q}}_p} \xrightarrow{\sim} \bigoplus_f V_f \otimes_{\overline{\mathbb{Q}}_p} \pi(V_f),$$

where $\pi(f) = \bigotimes_\ell' \pi_\ell(V_f)$ is the rest. tensor ~~of~~ over all primes of LLC applied to V_f (the ass. Gal. rep. to a cusp. newform), and f runs over all cusp. newforms defined over $\overline{\mathbb{Q}}_p$ of wt. k.

- Would like to have a similar story for p-adic LL. H^1_{par} is too small to play this role (which mirrors the fact that at $\ell = p$ LL only applies to p.ss reps.).

  It is expected the right global object is completed cohomology.
  Actually for the p-adic story it's backwards: outside $GL_2(\mathbb{Q}_p)$ no p-adic LLC exists yet, but we have a guess on the global story. One major desiderata for p-adic LLC is local-global compatibility.
- One can try to recover p-adic LLC through comp. coh. + assuming local-global compatibility.
- This is (to my knowledge) the motivation for the 6-author patching paper & a major motivation for the categorical p-adic LL conjectures.
- There are other reasons to care: relatedly comp. cohomology provides a representation theoretic ~~appr~~ construction of the eigencurve. But focus will be on links to p-adic LL.

§1: Definition and Setup, Basic Properties:

<u>Setup/Assumptions:</u>   • $\mathbb{A} := \mathbb{A}_F$.

- G alg. gp., • F n.f., • $(E, \mathcal{O}_E)$ local field ext. $\mathbb{Q}_p$. Actually I've probably assumed throughout that $F = \mathbb{Q}$.
- Let K be a tame level (i.e: compact open subgp.) of $G(\mathbb{A}_f)$, w/ $K =: K_p K^p$ the decomp into at p & away from p parts.
- Let $Y(K_f) := G(F) \backslash G(\mathbb{A}) / K_\infty^\circ A_\infty^\circ K_f$ our modular spaces, K_∞ maxml cpct of G_∞, A_∞ maximal torus, $(\cdot)^\circ :=$ conn. comp. of the identity.

Rmks:

- Abstract setup makes sense for really any coh. theory and tower of spaces w/ gp. actions.
  Focus will be on H^i_{et} and Shimura vars. as we really want Gal. actions.
  Everyth throughout is natural enough to commute w/ comparison things between Étale & sing..
  - Feel free to assume $G = GL_2/\mathbb{Q}$, $F = \mathbb{Q}_p$.
  - Will need $H^i_{par} := \mathrm{Im}(H^i_c \to H^i_{et})$ later. All results (I think) hold regardless of this choice. Check this?

Now we'll vary K_p, the level structure at p.

Defn (Completed Cohomology): $\tilde{H}^i(K^p) = \varprojlim_s \varinjlim_{K_p} H^i_{et}(Y(K_pK^p)_{\bar{\mathbb{Q}}}, \mathcal{O}/\varpi^s)$.

Rmks: 1) Note the order of lims. Perhaps not the obvious one.
2) View this as equipped w/ p-adic top., It is complete. So $\tilde{H}^i_E$ is a E-Banach space.

Basic Properties:

- It is a $G_{\mathbb{Q}}$ (assuming for simp. $\mathbb{Q}$ is the reflex field) $\times$ $G(\mathbb{Q}_p)$ rep:
  The $G(\mathbb{Q}_p)$ rep crucially relies on having passed to infinite p-level.
  It is defined as follows: Let $g \in G(\mathbb{Q}_p)$.
  $$Y(K_pK^p) \xrightarrow{\cdot g} Y((g^{-1}K_pg)K^p) \quad \text{is well-defined,}$$
  $$\text{i.e: } Y(gK_pg^{-1}K^p) \xrightarrow{\cdot g} Y(K_pK^p).$$
  This induces $H^i_{et}(Y(K_pK^p)_{\bar{\mathbb{Q}}}, \mathcal{O}/\varpi^s) \longrightarrow H^i_{et}(Y(gK_pg^{-1}K^p)_{\bar{\mathbb{Q}}}, \mathcal{O}/\varpi^s)$.
  gK_pg^{-1} contains another cong. subgp. K_p' that we can map H^i_{et} to.
  The action comes from assembling all of these.
- ~~The~~ Further there is a commuting action of $\pi_0 := G_\infty/G_\infty^\circ \cong K_\infty/K_\infty^\circ$, the component gp., acting on the right.
  Eg: For GL_2, this is $\cong \mathbb{Z}/2\mathbb{Z}$, acting via $\tau \mapsto -\bar{\tau}$ on the usual modular curve picture.
- $\tilde{H}^i$ is an admissible ($\tilde{H}^{i,K}$ is f.d. for K cpct open) $G(\mathbb{Q}_p)$ rep but very much not smooth.
  For GL_2: $(\tilde{H}^i)^{K_p} \cong H^i(Y(K_pK^p), \mathcal{O})$ [this follows from some folklore S.S. + some hom.-alg., see [Eme-ICM, pg 62]].

Summary: $\tilde{H}^i(K^p)$ is a $G_{\mathbb{Q}} \times G(\mathbb{Q}_p) \times \pi_0$ rep., admissible in the $G(\mathbb{Q}_p)$ part.
- We'll check this action is more or less Hecke, see pg ⑤ after introducing coefficients.

Relation to an alt. defn.:

Here's an obvious potential "alternate defn.":

Defn: $(H^i(K^p))$: $H^i(K^p) := \varinjlim_{K_p} H^i_{et}(Y(K_pK^p), \mathcal{O}) \left(= \varinjlim_{K_p} \varprojlim_s H^i_{et}(Y(K_pK^p), \mathcal{O}/\varpi^s)\right)$.
$$\hat{H}^i(K^p) := \widehat{H^i(K^p)}^p.$$

Rmks: 1) As Zijian pointed out, this defn. is less natural: you're completing twice. 3 limits is a lot to work w/!
2) ~~The~~ $\exists$ natural map $\hat{H}^i \longrightarrow \tilde{H}^i$, chase the lims.

One can relate them much closer:

Prop: $\exists$ SES: $0 \to \hat{H}^i \to \tilde{H}^i \to T_p H^{i+1} \to 0$

Pf: Take SES of constant sheaves $0 \to \mathcal{O} \xrightarrow{\cdot \varpi^s} \mathcal{O} \to \mathcal{O}/\varpi^s \to 0$.

Associated LES $\rightsquigarrow$

$$0 \to H^i(Y(K_pK^p), \mathcal{O})/\varpi^s \to H^i(\text{''}, \mathcal{O}/\varpi^s) \xrightarrow{\delta} H^i(\text{''}, \mathcal{O})[\varpi^s] \to 0$$

Now take $\varinjlim_s \varprojlim_{K^p}$. No $\varinjlim^1$ term as transitions on the left term are surj. ∎

- So $\tilde{H}^i$ also contains some mysterious torsion contribution.

Hochschild–Serre: Descent to finite level:

- We know $H^i_{par}(Y(K^pK_p)_{\overline{\mathbb{Q}}}, \text{coeffs})$ is interesting cause it relates to ~~~~ Gal. reps attached to modular forms.
- ~~At minimum $\tilde{H}^*$ should recover H^i~~
- At minimum $\tilde{H}^*$ should recover $H^*(Y(K^pK_p)_{\overline{\mathbb{Q}}}, \mathbb{Z}_p)$ at finite levels.

Thm: (Hochschild–Serre, [Em06]): $\exists$ SS:

although see [CE] for integral version statement.

$$E_2^{i,j} := H^i(K_p, \tilde{H}^j(K^p)) \Longrightarrow H^{i+j}(Y(K_pK^p), \mathcal{O}).$$

Rmks: • More generally there's a S.S. for taking smooth parts (which certainly covers finite level contributions) & this is ~~affectively~~ obtained by taking invariance of this. More generally taking vectors satisfying some p-adic FA conds. is interesting.

• See [Em06] for more (at least rationally). Pf is usual S.S. stuff, but ~~~~ on an explicitly constructed CX.

§2: Modular forms:

Equivariant V. bundles:

- To recover reps. ass. to modular forms, we need to attach v. bundles to $Y(K^pK_p)$, functorially in K_p's.

Setting: W is a f.g. $\mathcal{O}$-module, w/ a (cont) action of $K_0 \subseteq$ ~~GL~~ $G(\mathbb{Q}_p)$, K_0 cpct. open.

We now attach (adele-theoretically) a local system to $Y(K_pK^p)$ (for $K_p \subseteq K_0$) w/ fibers $\cong W$:

$$\mathcal{W} := G(\mathbb{Q}) \backslash \left(G(\mathbb{A}) / K_\infty^\circ A_\infty^\circ K^p \times W \right) / K_p.$$

- This is defined compatibly over all $K_p' \subseteq K_0$, so the following makes sense:

$\mapsto$ Defn: $\tilde{H}^n_*(K^p, \mathcal{V}_W) := \varprojlim_s \varinjlim_{K_p} H^n_*(Y(K_pK^p), \mathcal{W}/\varpi^s)$, $\tilde{H}^n_*(K^p, \mathcal{V}_W)_E = E \otimes_{\mathcal{O}_E} \tilde{H}^n_*(K^p;\mathcal{V}_W)$.

~~Indept.~~

$\llcorner$ If W is f.d. E-v.s. w/ a K_0-invt. lattice (which exist w/ suff. small K_0), we can define this wrt the lattice, and ~~the def is ~~~~~~ $\tilde{H}^n_*(K^p, \mathcal{V}_W)_E$ is indpt. of lattice choice.

E.g.: $W = \mathbb{Z}_p^2$ w/ ~~the action~~ $\underline{v} \cdot g_p := g_p^{-1} \cdot \underline{v}$ (Left action is the obvious $GL_2(\mathbb{Z}_p) \circlearrowright \mathbb{Z}_p^2$ act.).

$\llcorner$ One traces that $\mathcal{V}_{\mathbb{Z}_p^2} \cong R^1 pr_* \mathbb{Z}_p$ (both associate to a curve its Tate module).

$\llcorner$ ~~Earlier~~ Recall Chengyang's talk:

$$H^1_{par}(Y(K_f), \mathcal{V}_{\mathbb{Z}_p^2})_{\mathbb{Q}_p} \cong H^1_{par}(Y(K_f), R^1 pr_* \mathbb{Z}_p) \cong \bigoplus_{\substack{f \text{ wt } 2 \text{ cusp forms} \\ \text{level } K_f}} \rho_f.$$

maybe $\oplus$ its dual

So this v.b. ~~is the right~~ detects reps of modular forms.

E.g2: For wt $k \geq 2$ in general, one takes $W = \mathrm{Sym}^{k-2} \mathbb{Z}_p^2$ & has the same story.

Souped-up Hochschild–Serre:

Thm: W as above. $\exists$ s.s.:

$$E_2^{i,j} = \mathrm{Ext}^i_{\mathcal{O}[[K_p]]}(W^\vee, \tilde{H}^j_{(K^p)}) \Longrightarrow H^{i+j}(Y(K_pK^p), \mathcal{W}).$$

(see [EmICM14] ~~~~ §2.1.3) for the precise statement).

Rmks: • Note $\tilde{H}^j(K^p)$ knows nothing about W. So E.g: $\tilde{H}^1(K^p_p)$ captures info. on all wts (≥ 2) of modular forms. So at minimum it is big.

$G(\mathbb{Q}_p)$-action & Hecke operators:

The $G(\mathbb{Q}_p)$ action should be related to Hecke operators at p.

To relate them we'll introduce some notation for averaging, mirroring the defn. of Hecke operators.

Notational Setting: We'll work w/ Γ_1 level structure to simplify notation a bit.

Defn: Let $N_0 := \left\{ \begin{pmatrix} 1 & \mathbb{Z}_p \\ 0 & 1 \end{pmatrix} \right\} \subseteq GL_2(\mathbb{Q}_p)$.

Let $\pi_{N_0, m} := \int_{N_0} n \cdot m \cdot \underline{v} \, dn$, where $m \in GL_2(\mathbb{Q}_p)$, this is an operator acting on $\hat{H}^1_*(K_p)$, and the Haar measure dn is normalized s.t. measure of N_0 is $\underline{1}$.

↳ Concretely: if $N_0 \cdot m = \bigsqcup_{i=0}^{r-1} m_i \cdot K_p$, where $K_p \cdot \underline{v} = \underline{v}$, then $\pi_{N_0, m}(\underline{v}) = \frac{1}{r} \sum m_i \cdot \underline{v}$.

Recall: that
$$Y(K_p K^p) \cong GL_2(\mathbb{Q}) \backslash GL_2(\mathbb{A}) / T(\mathbb{Q})^+ SO_2(\mathbb{Q}) \cdot K_p K^p$$
$$\cong \underbrace{\left(SL_2(\mathbb{Z}) \cap K_f \right)}_{\text{Non-adelic level subgp.}} \backslash \mathcal{H} \times \left(\mathbb{Z} / \det(K_p K^p) \right)$$

* The $G(\mathbb{Q}_p)$ action acts on the finite part, while classical Hecke operators act at the ∞ place on $\left(SL_2(\mathbb{Z}) \cap K_f \backslash \mathcal{H} \right)$. So relating the 2 is tracing through the above.

The $G(\mathbb{Q}_p)$-action is pulling back a cohom. class through right mult by $\cdot g$.

~~For simplicity, lets restrict to trivial coefficients coeff. in modular~~

~~Let $c_f \in H^1_{par}(Y(K_p K^p)_{\overline{\mathbb{Q}}}, V_{\mathbb{Z}_p^2})$. This corr. to a modular form via the hol. 1 form of coeffs:~~

Let $f(\tau)$ be a wt. $k+2$ cuspform. Then through duality we may view the ass. cohomology class $c_f \in H^1_{par}(Y(K_f)_{\overline{\mathbb{Q}}}, V_{W_0})$ as the hol. 1 form w/ coeffs. $f(\tau) \cdot \underbrace{(\tau, 1)^k}_{\in \mathrm{Sym}^k} \cdot d\tau$.

What I will show is right mult. by $g_p \in GL_2(\mathbb{Q}_p)$ on an elt. $(\tau, 1) \in Y(K_p K^p)$ is equivalent to left mult on the infinite place w/ a similar matrix, say γ.

This shows $g_p^* c_f$ corresponds to a cuspform f' w/ $f'(\tau) = f(\gamma \tau)$. I won't bother tracing through the coeffs.

Demonstration: $(\tau, 1) \cdot g_p = (\tau, g_p)$.

Let $\gamma \in GL_2(\mathbb{Q}_p)^+$ be such that $\gamma g_p \in K_f$ (such a γ exists by approximation).

Then: $(\tau, g_p) \sim (\gamma \tau, 1)$.

E.g: Claim: $\pi_{N_0, \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}}$ corr. to the operator T_p.

Pf: $N_0 \cdot \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix} \cdot \Gamma_1(N) = \bigsqcup_{i=0}^{p-1} \begin{pmatrix} p & i \\ 0 & 1 \end{pmatrix} \Gamma_1(N)$. So $\pi_{N_0 \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}} c_f = \frac{1}{p} \sum_{i=0}^{p-1} \begin{pmatrix} p & i \\ 0 & 1 \end{pmatrix}^* c_f$. Let $g_i := \begin{pmatrix} p & i \\ 0 & 1 \end{pmatrix}$

Let $\gamma_i := \begin{pmatrix} 1/p & -i/p \\ 0 & 1 \end{pmatrix} \in GL_2(\mathbb{Q})^+$. Then $\gamma_i \cdot \begin{pmatrix} p & i \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in \Gamma_1(N)$.

Hence $g_i^* c_f$ corresponds to $\gamma_i^* f$, i.e: the fn. $(\gamma_i^* f)(\tau) := f(\gamma_i \cdot \tau) = f\left(\frac{\tau - i}{p}\right)$.

But $p^{-1} \sum_{i=0}^{p-1} f\left(\frac{\tau - i}{p}\right)$ is exactly the T_p operator / U_p op. for $\Gamma_1(N)$.

Other E.g: $\pi_{N_0, \begin{pmatrix} p & 0 \\ 0 & p \end{pmatrix}}$ corr. to $p^k \cdot \langle p \rangle$. $\pi_{N_0, \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}}$ for $(v, p) = 1$ corr. to $\langle v^{-1} \rangle$.

Can define the Hecke away from p if we take $\varinjlim_{K^p}$ too.

See [Em06, pgs 66-68] for details.

Note: Didn't mention $\hat{H}^1_E \cong \tilde{H}^1_E$ for modular curves

§3: Relation to p-adic LL:

5

- Will ~~we~~ want to see modular forms of all levels:

$$\hat{\tilde{H}}^n_*(V_W) := \varinjlim_{K^p} \hat{\tilde{H}}^n_*(K^p, V_W), \quad \text{likewise for } \hat{H}^n_*.$$

  ↳ Topology given by the "locally convex inductive limit top."; ignore this.

- Here statements are mostly known for $GL_2(\mathbb{Q}_p)$, and some for others. Conjectural in general.

Hierarchies of Conjectures:

As Zijian explained to me, it is helpful to think ~~of~~ that there is a tower of conjectures in order of strength.

Most optimistically: p-adic LLC exists.

Here you will also hope for local-global compatibility

Still very interesting: Breuil-Mezard

&& $R = \mathbb{T}$ statements

(at least philosophically): these should imply each other, which is exactly the approach Kisin used for F-M.

and perhaps one can see the bottom 2 as (weak?) evidence for p-adic LLC to exist.

So: Big Conj. 1: There exists a p-adic LLC. ~~Furthermore, it is~~

Furthermore, we would hope that:

Big Conj. 2 (LG compatibility): ~~Let V be~~ ~~Roughly:~~

Let V be cont., irred., odd, n.r. outside a finite set of primes Σ E-rep. of G_L.

Then: $$\mathrm{Hom}_{E[G_L]}\left(V, \hat{H}^{\mathrm{smth}}_E\right) \xrightarrow{\sim} B(V|_{E_p}) \otimes \bigotimes_{\ell \neq p}{}' \pi_\ell(V), \text{ as } G(\mathbb{A}_f) \text{ rep}$$

where I won't bother specifying ~~"smth"~~ "reflex field" the degree, B ~~and~~ is the p-adic LLC, π_ℓ is LLC.

- These are known for $G = GL_2$ for $G_{\mathbb{Q}}$ and $GL_2(\mathbb{A}_{\mathbb{Q}_p})$ ~~(but not~~ under ~~mild~~ technical hypotheses.
  (original pf. by Emerton needed TW conditions but maybe not necc. now?)
  + $\bar{V}$ abs irred.

~~R = T statements~~

~~Less optimi~~

- But this is really a situation where we understand the global picture (a lot) better.
  So we could try to reverse our approach and recover p-adic LLC from the global pic.
  So we can hope the completion is realized by closure in H_E, our global picture.

The standard approach to this is patching:

The output in this case is ~~hopefully~~ a e.g. $R_{\bar{r}}[\![K_p]\!]$-module M ~~whose fib~~ ($\bar{r}$ a mod p $G_{\mathbb{Q}_p}$ rep.),

w/ a $G(\mathbb{Q}_p)$-action ext. the K_p action whose fibers realize the p-adic LLC, ~~be~~ roughly meaning:

for a $G_{\mathbb{Q}_p}$ p-adic rep. ρ ~~coming~~ known to come from automorphic information, say corr. to an automorphic rep $\pi_\infty \otimes \pi_p$, w/ $\pi_p \hookrightarrow H^{q_0}(Y)_{\mathfrak{m}}$:

Do I need to assume this?

In general this is hard to make precise w/out knowing how to define $B(\cdot)$.
But if we assume $V|_{E_p}$ is pot. s.s., then it is conjectured that $B(V|_{E_p})$ is some kind of completion of some variant of LLC.

~~the p-adic~~ $B(\rho)$ should be the closure of $(V^\vee \otimes_E \pi_p)$ in ~~M~~ the fibre M_ρ
(up to some duals and what not. See [Em ICM, §3.2] for more precision).
Ofc for this to work, M needs to know smth about how to translate Gal. reps. to automorphic reps.
But this is exactly what completed cohom. is conjectured to do.
So one guess is M should be a patching of comp. cohom.
↳ This is my understanding of the aim of the 6-author paper (see also Scholze's ultrapatching simplification).

$R = \mathbb{T}$ statements:

~~The other~~ Another big conjecture one can make is that the Hecke action on $\tilde{H}^{\deg}_{\text{et}}(Y(K_p))$ sees ~~exactly~~ pretty much all ~~odd~~ representations, it can be expected to see.
This takes the form of a "big $R = \text{big } \mathbb{T}$" type theorem.

Defn: ($\mathbb{T}$): Fix a tame level K^p. Let $\Sigma_0 (\ni p)$ s.t.: for $\ell \notin K_0$: K_ℓ is ~~a~~ a (hyperspecial) maximal cpct subgp. [e.g.: O_2].
Let $\mathcal{H}_\ell := \mathcal{H}(G(\mathbb{Q}_\ell)//K_\ell, \mathcal{O})$, the spherical Hecke algebra.
Then $\mathbb{T}_\Sigma :=$ closure of $\mathcal{O}[\mathcal{H}_\ell]_{\ell \notin \Sigma}$ in $\longrightarrow \prod_i \prod_{K_p} \prod_W \operatorname{End}(H^i_{\text{et}}(Y(K_pK^p), W))$.
Fact: $\mathbb{T}_\Sigma$ is complete semilocal, i.e.: $\mathbb{T}_\Sigma \cong \prod_{\underline{m}} \mathbb{T}_{\underline{m}}$ a finite product (isom. also ~~as~~ a homeo).
Conj: $\mathbb{T}_{\underline{m}} \cong R_{\rho_{\underline{m}}}$, where $\rho_{\underline{m}}$ is the ass. rep. into ("$\bar{\mathbb{F}}$-valued pts. of the C-gp. of G" Some LL duality nonsense; e.g.: for modular curve ($G = GL_2$), this is $\cong GL_2$),
where $R_{\rho_{\underline{m}}}$ is def. ring w/ fixed Artin conductor away from p, no local hypotheses at p. ~~[illegible]~~
↳ Known for $G_{\mathbb{Q}}$ & GL_2 under technical hypotheses (TW cond.: $\rho_{\underline{m}}$ abs. irred over $G_{\mathbb{Q}}(\zeta_p)$ & some others, e.g.: oddness). [Böckle, ...]

Even w/out assuming this full conj., this allows us to make some guesses:
Conj 1: Each local factor $\mathbb{T}_{\underline{m}}$ is Noeth., reduced, ϖ-torsion free, of dim $= \dim B + 1 - \ell_0$. (rk of G_∞ − rk of $A_\infty K_\infty$) "defect wrt discrete series"
~~Making~~ Further assuming some conjectures on Eisenstein & tempered reps. leads to more conjs.:
Conj 2: ~~$\tilde{H}_i$~~ $\tilde{H}_i = 0$ if $i > q_0$, "cd" $\tilde{H}_{q_0} = \ell_0 \leftarrow q_0 = (d - \ell_0)/2$: first column of tempered range].

See [Em-ICM, §3] for more.