PROGRAM NOTES AND ABSTRACTS FOR WEEKS 1 AND 2

Abstracts are listed in order of the talks, starting in WEEK 1

Apprentice Program: Daniil Rudenko

We will cover a variety of topics in algebra, geometry and combinatorics. More details will be given Monday morning, June 15.

Greg Lawler

TITLE: Random walk and the heat equation

ABSTRACT: Two closely related topics are random walk and heat flow. One sees this by considering heat as consisting of a large (infinite number?) of "heat particles" all moving randomly and independently. I will present some of the mathematics that makes this rigorous. I start with random walk in the integer lattice and then use this to give a discrete model for heat flow. Analysis of this flow will use tools from linear algebra. In the second week I will consider the continuous analogues: the random walk becomes Brownian motion; the discrete heat equation because a partial differential equation (called, amazingly, the heat equation!), and the linear algebra argument is replaced with one using Fourier series.

The only prerequisites are a rigorous course in calculus (at the level of the 160s at University of Chicago) and some linear algebra. This course has been given before and the notes have been published in a book "Random Walk and the Heat Equation" published in the Student Mathematical Library series by the American Mathematical Society

Ewain Gwynne

TITLE: Random trees

ABSTRACT: A discrete tree is a connected graph with no cycles. I will explain the bijection between discrete trees with n edges and $2n$-step walks in the non-negative integers which start and end at the origin. I will then discuss the construction of the continuum random tree, the random metric space which describes the large-scale behavior of uniform random trees with n edges as n goes to infinity. I will end by discussing the random graph obtained by gluing two discrete random trees together, which is connected to some (seemingly very difficult) open problems in the theory of Liouville quantum gravity. I will try to keep background knowledge to a minimum — a basic understanding of probability and metric spaces at the introductory undergraduate level should be sufficient.

Maryanthe Malliaris

TITLE: [2025] Is randomness simple or is it complicated?

ABSTRACT: In this pair of lectures we will start by taking a look at the model-theoretic random graph as an interesting object in its own right, and then consider its relation to other natural mathematical objects [in the sense of the Diaconis-Malliaris paper "Complexity and randomness in the Heisenberg groups (and beyond)", available at https://nzjmath.org/index.php/NZJMATH/article/view/134]. Little prior knowledge will be assumed and most things will be defined.

Michael Barz

TITLE: Calculus mod 5

ABSTRACT: Algebraic geometry is the study of shapes defined by systems of polynomial equations. This might seem a very strange class of shapes to single out. However, one advantage of a polynomial equation like $y = x^2$ is that it makes sense over many different number systems – you can solve it over the complex numbers, you can solve it over the integers, or you could solve it in modular arithmetic. A remarkable phenomenon in algebraic geometry is that these different incarnations of the shape somehow know about each other: you can predict the topology of the shape over the complex numbers by knowing about its incarnations over finite fields, for example. We are going to try and explore basic notions in algebraic geometry, starting with the idea of the functor of points of an algebraic variety. We will have a strong bias towards exploring how to do calculus (in the guise of 'algebraic de Rham cohomology') over different number systems, and how these new notions of calculus relate to calculus over the complex numbers. No background in commutative algebra or algebraic geometry will be assumed, but we will assume the audience knows what a ring is, and is comfortable with standard topics in multivariable calculus and linear algebra.

Carmen Rovi

TITLE: Morse Theory: from height functions to homology.

ABSTRACT: Imagine a landscape slowly flooding with water. As the level rises, the flooded region changes its topology: a new lake appears where a valley floor goes under, two separate lakes suddenly merge when a mountain pass is submerged, a hilltop finally disappears beneath the surface. These events happen at precise moments: exactly when the water level passes through a local minimum, a saddle, or a local maximum of the terrain. Between such moments, the boundary of the flooded region deforms continuously, changing nothing essential. Morse theory is the mathematical framework that makes this picture rigorous and extends it from landscapes to smooth manifolds of any dimension.

In these lectures, we develop Morse theory from scratch, starting with the motivating examples of height functions on the sphere, the torus, and surfaces of higher genus. We introduce smooth manifolds and their critical points, a prove the Morse Lemma. The key idea of the course is the construction of Morse homology. Given a Morse function f on a closed manifold, we build a chain complex whose generators are the critical points of f and whose boundary maps count oriented gradient flowlines connecting them. We prove that the resulting homology groups are independent of all choices of function and gradient, giving a genuine invariant of the manifold. As a payoff, we compute this invariant for surfaces of every genus, and prove that the Euler characteristic equals the alternating sum of critical-point counts, and use Morse homology to distinguish infinitely many lens spaces L(p, q) from one another and from the 3-sphere.

Peter May

TITLE: [2025] An algebraic topology smorgasbord

ABSTRACT: [2025] What does it mean to say that finite posets and finite spaces are (almost) the same? And why are they closely related to simplicial complexes? And how does that raise questions in geometric topology? How can a conjecture about finite groups be a question about finite spaces? Speaking of groups, what is equivariant algebraic topology? What is stable homotopy theory? What is equivariant stable homotopy theory? How does it give a generalization of Galois theory? I will raise questions at many levels, some of which we will explore in coming weeks.

For 2026, I am also thinking of something that starts very simply and ends in work in progress: From symmetric monoidal categories (grade school category theory) to equivariant and multiplicative infinite loop space theory (modern algebraic topology closely related to algebraic geometry, through algebraic K-theory)

Sasha Razborov

TITLE: TBA

ABSTRACT: