PROGRAM NOTES AND ABSTRACTS FOR WEEKS 1 AND 2

Apprentice Program: Daniil Rudenko

We will cover a variety of topics in algebra, geometry and combinatorics. More details will be given Monday morning, June 10.

Probability and Analysis: from Greg Lawler

Here is a summary for the first week or two of probability and analysis talks.

Probability: Greg Lawler

TITLE: Random Walk and the Heat Equation

ABSTRACT: This will be a multi-week series that shows that two seemingly different notions — random walk and diffusion of heat — are essentially the same mathematically. I will discuss how to model this, first using discrete time and space, and then moving to continuous time and continuous space. Along the way, we will encounter a number of concepts in probability, linear algebra (diagonalization), differential equations, and Fourier series (a continuous analogue of diagonalization). The prerequisite for the lecture is undergraduate analysis (the 200 level analysis sequence at U Chicago), but the hope is that it will also be interesting for those who have more background than this in analysis and/or probability.

Analysis: Greg Lawler

TITLE: An Introduction to Measure Theory and Lebesgue Integration

ABSTRACT: Many students will be doing advanced reading in analysis, differential equations, or probability. The mathematical background for all of this is the idea of measure and Lebesgue integration. In the probability world these are probability and expectation. I am going to give a one week introduction to measure theory and Lebesgue integration focusing on the basic assumptions and definitions, trying to explain "why it works so well". I will also discuss the main terminology used. I probably will not have time to do proofs in detail, but I will do one or two so one gets a flavor of the subject. The starting question is ask yourself is: I know what the length of an interval is. Is there a way to extend the definition of length to all subsets of the real line so that it satisfies the properties we like? Indeed, what is the key property (hint: answer is countable additivity)?

Added later, for week 2:

In the probability and analysis talks this coming week, Greg Lawler will continue his 1:30 talks on Monday, but Professor Ewain Gwynne will give three talks (2:45 Monday, 1:30, Friday and 2:45 Friday. Here are his title and abstract. This may be of interest to people from many different groups in the REU.

TITLE: Random trees

ABSTRACT: A discrete tree is a connected graph with no cycles. I will explain the bijection between discrete trees with n edges and $2n$-step walks in the non-negative integers which start and end at the origin. I will then discuss the construction of the continuum random tree, the random metric space which describes the large-scale behavior of uniform random trees with n edges as n goes to infinity. I will end by discussing the random graph obtained by gluing two discrete random trees together, which is connected to some (seemingly very difficult) open problems in the theory of Liouville quantum gravity. I will try to keep background knowledge to a minimum — a basic understanding of probability and metric spaces at the introductory undergraduate level should be sufficient.

Algebra and Geometry Tuesday June 11: Shmuel Weinberger

TITLE: The Riemann hypothesis for polynomials over a finite field

ABSTRACT: There's an analogy between prime numbers and polynomials over a finite field that motivates a great deal of modern number theory. I'll explain how to prove a statement for the latter that would imply the Riemann hypothesis in the integer case. It will also show how to construct finite fields if you don't know about them already.

TITLE: Surfaces and maps.

ABSTRACT: Riemann surfaces are surfaces and can be associated to polynomials over C (rather than finite fields). I'll talk about maps between Riemann surfaces, and deduce something that resembles Fermat's last theorem from topological considerations. The talk won't be fully self-contained, but I'll only ask you to believe things that aren't hard, but do take time to prove. (So it should be viewed as a motivational speech, rather than a math lecture.)

Algebra Thursday June 13: Justin Campbell

TITLE: Representations of a finite general linear group

ABSTRACT: Representation theory sits at the intersection of many different fields within mathematics, including number theory, algebraic and differential geometry, algebraic topology, and functional analysis. It also plays a central role in quantum mechanics, where the state space of any system forms a representation of its symmetries. The interdisciplinary nature of representation theory can make it difficult to learn, since the subject comes in many flavors, each of which relies on its own set of techniques.

In these talks, I'll try to convey the big picture of representation theory as much as possible while focusing on the specific example of invertible 2x2 matrices with entries in a finite field. We'll be able to see many of the general patterns in this case with minimal technical complications. The talks should be understandable to anyone with a solid background in linear algebra.

Geometry: Thursday June 13: Ben Lowe

TITLE: Systolic geometry

ABSTRACT: The lectures I give will be an introduction to some ideas in systolic geometry. I will work towards proving the following statement, which relates length and volume given knowledge of the topology of the ambient space: for an n-dimensional torus with a Riemannian metric of unit volume, there is some noncontractible loop of length at most some universal constant depending only on n.

The goal will be to build intuition by thinking about examples (starting off in the 2-d case where the theory of Riemann surfaces is crucial), and to show how the previous statement connects to a range of important ideas in geometry, topology, and the calculus of variations.

Logic: Maryanthe Malliaris

TITLE: Model theory and limit points

ABSTRACT: The model theoretic way of looking at an infinite structure naturally gives rise to families of limit points, called "types," which can relate to those familiar from other areas of mathematics (such as Dedekind cuts over the rationals) but also have their own special character. This series of talks will be a gentle introduction to this very interesting idea and some of the things that can be done with it. No prior knowledge of model theory will be assumed.

Algebraic Topology: Peter May

Two topic series, one more elementary than the other. The titles are the same as last year, but the content is not.

Title: Finite spaces and larger contexts

Abstract: A finite space is a topological space with finitely many points. Finite spaces are "isomorphic" to finite posets and "equivalent" to finite simplicial complexes. They relate well to categories, simplicial sets, and general topological spaces. They are entering the applied world through data analysis and discrete Morse theory, and they are intrinsically related to many areas of current mathematical interest. We will start slow and go as far as we can. As an easy miracle, we will see a space with six points and infinitely many non-zero homotopy groups. This is based on a book in progress which now has much material that is not in the version posted last year. THIS SERIES SHOULD BE ACCESSIBLE TO APPRENTICES, although most of the material is likely to be new to those more advanced, even those with background in algebraic topology.

Title: Operads and iterated loop spaces

Abstract: This is an area a half century old that is undergoing current reinvestigation, both on an elementary combinatorial level, which we shall emphasize, and on a more abstract and yet quite concrete level. We will explain new concepts of "multiplicative operads" and "bioperads" and how they arise in nature. The starting point could easily have been developed a half century ago since it starts from simple structures that could and should have been understood then. There are many open problems coming from the new material. It is quite possible that the new operadic structures will have as many applications unrelated to my own motivation as the original operadic structures have had. My own motivation comes from equivariant multiplicative infinite loop space theory, one starting point for modern "derived algebra", in algebraic topology and algebraic geometry, but I have no idea how far we might get into that.

Special lecture: Matthew Emerton

Title: Studying numbers, from ancient Greece to modern times

Abstract: This lecture, which presumes just a high school background in mathematics, will present a high-level overview of the study of algebraic numbers.

Beginning with the theory of geometric constructions from ancient Greek geometry, and its relationship to the discovery and properties of irrational numbers, I will sketch in broad outlines how these ideas evolved, through the theory of equations and their symmetries as developed by Galois, culminating in a (very) brief description of some of the contemporary aspects of the theory.