

ABSTRACTS FOR WEEKS 7 AND 8

Probability and Analysis, continuing: from Greg Lawler:

Probability: I will finish a description of all Levy processes (infinitely divisible distributions) and then will finish my lectures with an introduction to some of the random fractals that appear in probability and statistical physics.

Analysis: Talks by Selma Yildirin:

TITLE: On hearing the shape of drums

ABSTRACT: In 1966, M. Kac published a paper with the title “Can one hear the shape of the drum?”, which inspired many works not only in Mathematics, but also in a wide range of other areas. During the first talk, we will see what Kac means with this question, go over related history and explore other ideas inspired by this question. In the second talk, we will explore some examples showing that we cannot hear the shape of the drum and learn more about what info that we can get and when. The final talk will focus on some related work and the connection with symmetric stable processes.

Quantative topology: Shmuel Weinberger

TITLE: Introduction to Persistent Homology, Variational Problems, and Quantitative Topology

Abstract: Quantitative topology focuses on the “sizes” of objects that topology studies. Such as: how much deformation is necessary in the course of homotoping one function to another. Variational problems are basically about calculus on function space: ”what route from A to B (through a perhaps inhomogeneous medium) takes the least time?” is the formulation of Fermat’s principle in optics – but many problems in math and physics are of a variational nature. Persistent homology is a refinement of homology theory that was introduced originally as a tool in applied math (for understanding the functioning of proteins and for computer mapping of snow fields) that is useful in this setting (and indeed has become a standard tool in a bunch of different areas of math from symplectic topology to spectral geometry and elsewhere). I will explain background about homology, morse theory, persistent homology, etc. and use this to explain a theorem of Gromov about closed geodesics on certain manifolds.

Special talk. Category theory: Eugenia Cheng

TITLE: Associativity, Commutativity and Units: a Higher-dimensional ballet

ABSTRACT: Associativity, commutativity and unit laws are axioms we typically learn about early on, in the context of numbers. We might then take them for granted until we meet non-commutative situations, such as multiplication of matrices, or symmetry groups. In higher dimensions we start to encounter non-associativity and non-unitality as well, but there is more nuance: rather than associativity simply being true or not true, there are shades of gray, where associativity holds up to isomorphism, equivalence, or just some sort of map. In this talk I will describe how those familiar three families of axioms become the essence of all the interesting features of weak higher-dimensional category theory. Moreover, rather than being three different types of axioms they are inextricably related via a higher-dimensional version of distributivity. The ballet they present is one of ebb and flow, give and take, where rigidity for one “dancer” always needs to be offset by flexibility in another. I will show that the apparently mundane math of high school has deep category theoretical insights embedded in it, if we care to look. I will not assume any prior knowledge of category theory.

Continuing: Algebraic Topology: Peter May

TITLE: Operads and iterated loop spaces

ABSTRACT: This is an area a half century old that is undergoing current reinvestigation on a more abstract and yet quite concrete level. We will explain the interest of higher homotopical structure and show how simply it can be incorporated into elementary structures which hide the homotopies conceptually. Spectra and stable homotopy theory will be introduced. The focus will be on the process of constructing iterated loop spaces and spectra from structured spaces and categories, getting into equivariant and multiplicative contexts as and if time permits.

TOWARDS THE END: Probably some work in progress, depending on progress: Operads, Moperads, and Bioperads. New takes on permutative and bipermutative categories. Equivariantly a description of permutative G -categories and ideas on what bipermutative G -categories are. This may be a categorical follow-up to Eugenia Cheng’s talk and it may be a preview of a workshop in Beijing in August.