ABSTRACTS FOR WEEKS 3 AND 4

Apprentice Program: Daniil Rudenko and/or Laci Babai

Continuing: We will cover a variety of topics in algebra, geometry and combinatorics.

Probability and Analysis, continuing: from Greg Lawler

Here is a summary for the probability and analysis lectures.

I will be giving two independent but complementary lectures in the Probability and Analysis section. One can easily attend only one, but those who attend both will see that they are different approaches to the same general idea. There is no strict border between probability and analysis as subjects — indeed, I will introduce Brownian motion in the analysis part but I may build on this in later weeks in the probability part.

TITLE: Markov chains and Laplacians

ABSTRACT: In probability, we will consider a basic discrete model, Markov chains, but focus on more sophisticated aspects than might be considered in a first course. In particular, we will look at the Green's function, Laplacian, and discrete "heat equations". A particular example that will be considered is simple random walk on the integer lattice. No previous knowledge of Markov chains is needed but I will try to discuss different material than I do in Math 235.

TITLE: Harmonic functions, heat equation, and the Laplacian

ABSTRACT: We will start with the idea of a harmonic function as a function satisfying the "mean value property" and this will lead to the Laplacian which can be considered as the "rate of change of the mean value". The notion of mean value naturally leads to the idea of "random continuous motion", that is, Brownian motion. We will discuss some of the standard elliptic and parabolic PDEs such as the Laplace equation, heat equation, and Poisson equation. In future weeks, we will discuss some topics from functional analysis and Fourier analysis but it is good to understand how the latter subjects are (at least partially) motivated by problems in differential equations.

Probability: Ewain Gwynne

TITLE: Introduction to percolation

ABSTRACT: Let $p \in [0,1]$, and suppose that we independently color each edge of the square grid red with probability p and blue with probability $1-p$. For which values of p does the set of red edges have an infinite connected component? For such values of p, is there only one connected component or many? Questions of this type (and their analogs on other graphs) are the topic of percolation theory. I will give an introduction to this theory, which will include answers to the above questions for the square grid, as well as some discussion of open problems for percolation in higher dimensions. I will assume only a basic understanding of probability at the introductory undergraduate level.

Continuing: Geometry: Aaron Calderon and Ben Lowe

TITLE: Hyperbolic geometry and low-dimensional topology

ABSTRACT: Hyperbolic geometry is of fundamental importance in low-dimensional topology. We will give a crash course on hyperbolic geometry and then move to some topics of current interest, possibly including the connection between circle packings and Kleinian groups.

Continuing: Algebraic Topology: Peter May

Two topic series, one more elementary than the other.

Title: Finite spaces and larger contexts

Abstract: A finite space is a topological space with finitely many points. Finite spaces are “isomorphic” to finite posets and “equivalent” to finite simplicial complexes. They relate well to categories, simplicial sets, and general topological spaces. They are entering the applied world through data analysis and discrete Morse theory, and they are intrinsically related to many areas of current mathematical interest. We will start slow and go as far as we can. As an easy miracle, we will see a space with six points and infinitely many non-zero homotopy groups.’

Title: Operads and iterated loop spaces

Abstract: This is an area a half century old that is undergoing current reinvestigation on a more abstract and yet quite concrete level. We will explain the interest of higher homotopical structure and show how simply it can be incorporated into elementary structures which hide the homotopies conceptually. Spectra and stable homotopy theory will be introduced. The focus will be on the process of constructing iterated loop spaces and spectra from structured spaces and categories, getting into equivariant and multiplicative contexts as and if time permits.

Continuing: Logic: Maryanthe Malliaris

TITLE: Cardinal invariants of the continuum

ABSTRACT: Cardinal invariants of the continuum give an interesting way of studying what was, early last century, the conjecturally nonempty region between aleph-1 (the first uncountable cardinal) and the continuum. Even after Cohen’s invention of forcing, they continue to open up many subtle questions about infinity. In these lectures, we will start more or less from scratch and take a short walk in this world.

Number theory: Zijian Yao

TITLE: Elliptic curves with complex multiplication

ABSTRACT: In these two lectures I will try to explain what elliptic curves are, the theory of complex multiplication, and their connections to Galois theory (in particular with class field theory), (and hopefully to) modular forms and L-functions.

Complexity: Alexander Razborov

TITLE: Mysterious Simulations

ABSTRACT: Developing methods to rigorously separate different classes of algorithms and computational models from each other is the Holy Grail of the classical complexity theory. The most famous problem of this kind, aka the P vs. NP question, is whether non-deterministic computations can be efficiently reduced to deterministic ones; there are many others. The rule of thumb is that if there does not appear to exist any good reasons for inclusion (of one complexity class into another) then most likely it does not exist, and the best bet is that actually proving this is a grand open problem well beyond the reach of current methods. In this series of lectures we will discuss a few beautiful and unexpected results that go against this common wisdom and establish efficient simulations of one class of algorithms by another; they are rather transcendental in their nature. When feasible, we will give either a complete proof or at least a sketch.

REFERENCES:

General Reading

1. S. Arora, B. Barak, Computational Complexity: a modern approach, Cambridge University Press, 2009.
2. S. Jukna, Complexity of Boolean functions, Springer-Verlag, 2012.
3. R. O'Donnell, Analysis of Boolean functions, Cambridge University Press, 2014.
4. P. Hatami, R, Kulkarni, D. Pankratov, Variations on the Sensitivity Conjecture, Theory of Computing Library Graduate Surveys 4 (2011), pp. 1-27.

Concrete Results

[Jukna, Ch. 15,4], [Jukna, Theorem 14.3] and
5. H. Huang, Induced subgraphs of hypercubes and a proof of the sensitivity conjecture, Annals of Mathematics, 190(3), pp. 949-955 (the proof itself is 2 pages).

Representation theory: Justin Campbell

TITLE: Representation theory and Weil's Rosetta Stone

ABSTRACT: Representation theory is a broad web of interconnected results and heuristics, which spans fields as seemingly disparate as number theory, algebraic geometry, and quantum mechanics. In these talks I will attempt to convey some of the flavor of this vast subject.

Geometry-Topology: Carmen Rovi

Thursday TITLE: An introduction to Topological Quantum Field Theories

ABSTRACT: Historically, there has been a strong connection between geometry, topology, and physics. Topology provides a good framework to formalize certain quantum phenomena mathematically. In this lecture I will focus on the mathematical side of this story and will give an introduction to 2-dimensional Topological Quantum Field Theories (TQFTs). The main ingredients of the lecture will be manifolds, cobordisms and some category theory. I will attempt to make the lecture accessible and students of all levels are welcome.

Friday TITLE: Topology meets Physics: Scissors congruences and TQFTs.

ABSTRACT: Topology is sometimes referred to as "rubber-sheet geometry", and like geometry, it is concerned with the study of spaces. Among the most interesting spaces are "manifolds". Manifolds are sets of points locally modeled on Euclidean space. In this talk, we will explore the notion of cutting and pasting of manifolds. It turns out that these cut-and-paste operations determine interesting algebraic structures, which have strong connections to Topological Quantum Field Theories.