PROGRAM NOTES AND ABSTRACTS FOR WEEK 2

Apprentice Program: Daniil Rudenko

Continuing: We will cover a variety of topics in algebra, geometry and combinatorics.

Probability and Analysis: from Greg Lawler

Here is a summary for the probability and analysis lectures for the second week.

I will be giving two independent but complementary lectures in the Probability and Analysis section. One can easily attend only one, but those who attend both will see that they are different approaches to the same general idea. There is no strict border between probability and analysis as subjects — indeed, I will introduce Brownian motion in the analysis part but I may build on this in later weeks in the probability part.

TITLE: Markov chains and Laplacians

ABSTRACT: In probability, we will consider a basic discrete model, Markov chains, but focus on more sophisticated aspects than might be considered in a first course. In particular, we will look at the Green's function, Laplacian, and discrete "heat equations". A particular example that will be considered is simple random walk on the integer lattice. No previous knowledge of Markov chains is needed but I will try to discuss different material than I do in Math 235.

TITLE: Harmonic functions, heat equation, and the Laplacian

ABSTRACT: We will start with the idea of a harmonic function as a function satisfying the "mean value property" and this will lead to the Laplacian which can be considered as the "rate of change of the mean value". The notion of mean value naturally leads to the idea of "random continuous motion", that is, Brownian motion. We will discuss some of the standard elliptic and parabolic PDEs such as the Laplace equation, heat equation, and Poisson equation. In future weeks, we will discuss some topics from functional analysis and Fourier analysis but it is good to understand how the latter subjects are (at least partially) motivated by problems in differential equations.

Number Theory: Matt Emerton

TITLE: Special values

ABSTRACT: I will discuss some special value formulas from number theory, including the famous formulas

$$1 + 1/4 + 1/9 + \ldots = \pi^2/6$$

and

$$1 - 1/3 + 1/5 - \ldots = \pi/4$$

I will explain some of the history behind them, and their significance, and introduce some methods for proving them.

Continuing: Geometry: Aaron Calderon and Ben Lowe

TITLE: Hyperbolic geometry and low-dimensional topology

ABSTRACT: Hyperbolic geometry is of fundamental importance in low-dimensional topology. We will give a crash course on hyperbolic geometry and then move to some topics of current interest, possibly including the connection between circle packings and Kleinian groups.

Continuing: Algebraic Topology: Peter May

Two topic series, one more elementary than the other

Title: Finite spaces and larger contexts

Abstract: A finite space is a topological space with finitely many points. Finite spaces are "isomorphic" to finite posets and "equivalent" to finite simplicial complexes. They relate well to categories, simplicial sets, and general topological spaces. They are entering the applied world through data analysis and discrete Morse theory, and they are intrinsically related to many areas of current mathematical interest. We will start slow and go as far as we can. As an easy miracle, we will see a space with six points and infinitely many non-zero homotopy groups.

Title: Operads and iterated loop spaces

Abstract: This is an area a half century old that is undergoing current reinvestigation on a more abstract and yet quite concrete level. We will explain the interest of higher homotopical structure and show how simply it can be incorporated into elementary structures which hide the homotopies conceptually. Spectra and stable homotopy theory will be introduced. The focus will be on the process of constructing iterated loop spaces and spectra from structured spaces and categories, getting into equivariant and multiplicative contexts as and if time permits.

Logic: Maryanthe Malliaris

TITLE: Cardinal invariants of the continuum

ABSTRACT: Cardinal invariants of the continuum give an interesting way of studying what was, early last century, the conjecturally nonempty region between aleph-1 (the first uncountable cardinal) and the continuum. Even after Cohen's invention of forcing, they continue to open up many subtle questions about infinity. In these lectures, we will start more or less from scratch and take a short walk in this world.