ABSTRACTS FOR SECOND WEEK PROGRAM, JUNE 29–JULY 3

1:00 CENTRAL STANDARD TIME, Monday, Wednesday, and Friday

GREGORY LAWLER

PROBABILITY, RANDOM FIELDS, AND GEOMETRY IN STATISTICAL PHYSICS

ABSTRACT (from first week): We will be looking at models that arise in critical phenomena in statistical physics. The general framework is that there is a collection of sites and there is a random "field" defined on the sites. This field can be either a collection of random variables indexed by the sites or a random path or subgraph.

The lectures will focus on two main examples: the loop-erased walk which is closely related to uniform spanning trees and the Gaussian field. The lectures discuss the relationship of these to Markov chains and usual random walks, "loop measures". and determinants of the Laplacian. There are other models that participants may consider such as Ising model, percolation, and Potts models.

Other participants may consider the continuous analogues of these fields and random curves, and, in particular, the Schramm-Loewner evolution and the definition of the determinant of the Laplacian in the continuum. Other possibilities are random geometry (quantum gravity) and physics approaches to conformal field theory.

We will use facts in undergraduate mathematics from the following areas: linear algebra, (post-calculus) probability, real variables, complex variables, combinatorics and graph theory. These should not be considered strict prerequisites but they give hints to outside reading that participants may have to do.

The mathematics in the discrete models will involve a lot of combinatorics and should be of interest for those who like this kind of mathematics. The continuous analogues involve a lot of analysis (real, complex, and stochastic) and some PDE. It is not required to have much background in this; indeed, many find learning the discrete theory to be a good start before attacking the mathematically more sophisticated continuous theories.

It is hoped that the lecture notes for this class will become a book.

1:00 CENTRAL STANDARD TIME, Tuesday and Thursday

FOCUS GROUPS
Send email to host to request access

GREGORY LAWLER; lawler@math.uchicago.edu

PROBABILITY

PETER MAY; may@math.uchicago.edu

ALGEBRAIC TOPOLOGY

Wednesday July 1, 10:00 CENTRAL STANDARD TIME (special day and time)

MATTHEW MORROW

PERFECTOID SPACES

ABSTRACT: P-adic arithmetic geometry is concerned with questions in number theory and algebraic geometry which focus on a particular prime number p, such as the study of equations up to congruences modulo all powers of p and the associated symmetries (Galois groups). The subject has seen enormous progress in recent years, largely thanks to Peter Scholze?s introduction of "perfectoid spaces??, through which such questions may be "tilted?? into a more manageable world of geometry where $p = 0$. In the talk I will attempt to present some of the main ideas around perfectoids and tilting. Familiarity with rings, polynomials, and field extensions will be assumed.

MWF, June 29, July 1, July 3 2:30 CENTRAL STANDARD TIME

INNA ZAKHAREVICH

SCISSORS CONGRUENCE

ABSTRACT: The ancient Greeks defined area as "that which does not change under cutting and moving." In this series of talks we will take this definition, and its extension to other geometries and dimensions, seriously: examining it's implications, investigating new invariants it gives rise to, and exploring its connections to other fields of mathematics. Given time and interest, we will introduce homological algebra for geometric objects using the perspective of scissors congruence.

Tuesday, June 30 2:30 CENTRAL STANDARD TIME

DANIEL LITT

COMBINATORICS AND THE LIE ALGEBRA sl_2

ABSTRACT: Let $d_{n,k}$ be the number of isomorphism classes of graphs with n vertices and k edges. We don't know a formula for $d_{n,k}$, or even an efficient algorithm for computing it. Nonetheless, I'll describe a proof that if one fixes n and varies k, the sequence $d_{n,k}$ is unimodal (that is, it has only one "peak"). The proof uses the representation theory of the Lie algebra sl_2, which I will introduce and develop from scratch.

Thursday, July 2 2:30 CENTRAL STANDARD TIME

EUGENIA CHENG

CATEGORY THEORY AND LIFE

ABSTRACT: Category theory can be thought of as being "very abstract algebra". It is thought of as "too abstract" by some people, and as "abstract nonsense" by some others. In this talk I will show that while it is abstract, it is far from being nonsense. I will argue that the abstraction has a purpose and that broad applicability is one of the powerful consequences. To demonstrate this, I will show how I apply concepts of category theory to important questions of life such as prejudice, privilege, and equality. I will introduce the category theory concepts from scratch so no prior knowledge is needed. These concepts will include objects and morphisms, isomorphisms and universal properties.

4:00 CENTRAL STANDARD TIME

AKHIL MATHEW; Tuesday and Thursday

QUADRATIC FORMS

ABSTRACT: I will discuss some aspects of the arithmetic of quadratic forms, including their classification over finite, local, and global fields, and the connection to more recent developments.

PETER MAY; Monday, Wednesday, and Friday (also Tuesday, Thursday 1:00 focus groups)

TOPICS IN AND AROUND ALGEBRAIC TOPOLOGY

For those new to algebraic topology here is an historically oriented introductory talk YouTube

Topic 1: FINITE SPACES AND LARGER CONTEXTS

It is a striking, if esoteric, fact that in principle one can do all of algebraic topology using Alexandroff spaces, in which arbitrary intersections of open sets are open. That is even true equivariantly, when one considers spaces with symmetries given by group actions. Alexandroff spaces in which the topology distinguishes points are the "same thing" as partially ordered sets (posets). This establishes a close connection between algebraic topology and combinatorics. There are many applications, some speculative, others far out. For example, finite spaces have been used to study the role of RNA in evolution. I am writing a book on this subject Finite spaces and I am presenting topics from it and from past REU papers (maybe some of you might want to contribute).

Topic 2: CLASSIFYING SPACES AND CHARACTERISTIC CLASSES

This is classical algebraic topology that everyone should know but is seldom taught. The passage from geometric topology to algebraic topology passes through the equivalence of vector bundles with homotopy classes of maps into classifying spaces. Then characteristic classes are invariants of bundles that are determined by the cohomology of classifying spaces. This is also the subject of a book in progress Characteristic classes

Topic 3: EQUIVARIANT HOMOTOPY AND COHOMOLOGY THEORY

What is equivariant algebraic topology? In particular, what are equivariant cohomology theories and what are they good for? This is the subject of the book Equivariant homotopy and cohomology theory and of the talk Equiv

Topic 4: OPERADS, OPERAD PAIRS, AND THEIR ALGEBRAS

Modern abstract homotopy theory largely evolved from early work on iterated loop space theory: A_∞-spaces, E_n-spaces, and E_∞-spaces, and the spectra associarted to the last. This is a more advanced topic, but I may give it a try, focusing on the relationships among spaces, categories and "stable spaces" or spectra. Expository papers include What?, How?, and Why?.