# NOTES ON THE 2017 REU PROGRAM

Here are some notes about the program. They will be updated throughout the program as more information comes in. The program is frontloaded, with the number of talks decreasing as the focus switches to your work on papers.

APPRENTICE PROGRAM: László Babai

Weeks 1-5. I do not have an abstract for the complete 2017 Apprentice program, but let me refer you to Professor Babai's web page about past years give an idea.

http://people.cs.uchicago.edu/ laci/reu

To quote from 2016: The apprentice course will be more structured than in the past. In order to allow it to reach serious conceptual foundations and interesting applications in quasi-polynomial time (google "quasipolynomial time"), all apprentice participants will be expected to genuinely participate by turning in problem sets at prescribed times, and they will be given feedback on them. REU participants in the full program are also welcome to participate; there will be ample material in the course they will probably not have seen before.

Group work will be encouraged on more complex problems, while students will be expected to solve rudimentary problems individually. Apprentice papers are strongly encouraged to relate to the apprentice course. Professor Babai will prepare guidelines and proposed topics for these papers. One option will be to develop solutions to challenge problems or problem sequences into a coherent exposition. As stated in the announcement of the program, "apprentice papers not related to the lectures must be approved by the program director."

FULL PROGRAM: There will be sequences of talks in many areas. Some will last two or three weeks, others longer, others shorter; some changes from the following are possible, although the first few weeks are pretty much set.

PROBABILITY: Weeks 1 and 2: Gregory Lawler

Problems and Paradoxes in Probability

I will consider some well known problems (some of which are called paradoxes) and explain how they are analyzed. Along the way I will be presenting some important concepts and structures in probability, I will treat one or two problems a day so each lecture can be considered a separate presentation. Among the things I will discuss are: hat problem (random permutations), secretary problem. St. Petersburg paradox, Monte Hall problem, two envelopes problem, Polya's urn and relation to Bayesian inference.

These lectures should be appropriate for all undergraduates whether or not they have had a course in probability.

GEOMETRY: Weeks 1 and 2: Shmuel Weinberger

Groups graphs and geometry

Abstract: Finitely generated groups are actually geometric objects once one decides what geometry means. I plan to take a random walk through the vast subject connecting manifolds and groups both geometrically and analytically. (Slogan: although subjects exist, the boundaries between them do not.)

ALGEBRA AND NUMBER THEORY: Weeks 1 through 4 Frank Calegari, Matt Emerton, Jack Shotton

Title: patterns amongst primes

(maybe "among" for a US audience?)

Abstract: We discuss some classical problems about prime numbers. Included: primes of the form $x^2 + Dy^2$ for various values of $D$, and Dirichlet's theorem on primes in arithmetic progressions,.

LOGIC: Week 1 Denis Hirschfeld

Title: Computable Mathematics and Reverse Mathematics

Abstract: We are often interested in questions of implication between true statements, when we say things like "Theorems A and B are equivalent" or "Theorem C does not just follow from Theorem D". These are often crucial things to understand about an area of mathematics, and can also help us make connections between different areas. Computability theory and proof theory can both be used to analyze, and hence compare, the strength of theorems and constructions. For example, when we have a principle such as "Every infinite binary tree has an infinite path", we can ask how difficult it is to compute such a path from a given tree. We can also ask how much axiomatic power is necessary to prove that this principle holds. The first kind of question leads to the program of Computable Mathematics. One version of the second kind leads to the program of Reverse Mathematics. I will give an introduction to these research programs, and discuss how close connection between computability and definability yields a fruitful interplay between them.

A version of a recent book on these topics is available here:
http://math.uchicago.edu/ drh/Papers/Papers/rm.pdf

LOGIC: Weeks 4 through 6: Maryanthe Malliaris

Title: Ultrapowers of linear order

Abstract: Starting from a basic construction – an ultrapower of linear order – we will arrive at current research. I plan to give most definitions and many proofs.

ALGEBRAIC TOPOLOGY: Weeks 1 through 8: Peter May and Dylan Wilson

We plan tentatively to organize the course around interesting theorems and problems. For example, we might begin by introducing the 'hairy ball theorem", or why you can't comb your hair without a cowlick. That gives an excuse for introducing vector bundles, vector fields, Euler characteristic, degrees of maps, and the Poincare-Hopf theorem (which shows how to compute an analytic invariant as a topological invariant). We intend to give some historical context and to highlight relationships between algebraic topology and related areas of mathematics, in line with Shmuel Weinberger's slogan. Since this series of talks will run through the entire program, we shall try very hard to make it accessible to all and yet interesting to those who have already seen some algebraic topology.

DYNAMICS: Week 2: Aaron Brown

Title: Ideas in dynamical systems

Abstract: We will discuss some ideas from the theory of dynamical systems motivated by examples of two families of circle dynamics: circle rotations and integer multiplication. We will explore topological properties (denseness of orbits, existence of exotic invariant sets) and statistical properties (ergodicity, equidistribution, mixing) through these two examples.

ANALYSIS: Weeks 3 and 4: Charles Smart

Title: Mean curvature flow

Abstract: We will study motion by mean curvature – the evolution of soap bubbles. We will begin with numerical methods and the level-set formulation. We will then discuss phenomena such as self-shrinkers, regularization, and fattening.

ANALYSIS: Weeks 4, 5, and 6: Will Feldman, Chris Henderson, Stanley Snelson

Title: Introductory topics in PDE

Abstract: We will discuss basic questions in the theoretical study of partial differential equations (PDE). Topics will include calculus of variations (e.g. the isoperimetric problem and minimizing Dirichlet's energy), spectral theory (eigenvalues of differential operators by analogy with symmetric matrices), and connections to probability (e.g. the heat equation and random walks). We may cover other related topics depending on time constraints and student interest. Lectures will be mostly independent from each other and will require little background knowledge.

SPECIAL: Friday Weeks 1 and 2: Andre Neves

The intersection between Analysis, Geometry, and Topology that happened in the last century is one of the great chapters in modern Mathematics. I will explain how the question of finding the least possible number of critical points for a function on projective space is related to cup product in topology and to the existence of simple closed geodesics on ellipsoids.

In the first lecture we will see why every function on a circle, 2-projective plane, and $n$-projective space has 2, 3 and $n+1$ critical points, respectively. In the second talk we will see how that can be used to find 3 simple closed geodesics on any 2-sphere.

SPECIAL: Wednesday Week 1: Eugenia Cheng

Deep Structures: an exploration of abstract mathematics

I will explore mathematics as a way of thinking. I will show some unexpected connections between examples in music, juggling, and baking, by uncovering abstract structures deep inside them. The examples will give a curious window into group theory, topology and category theory that can be shown to and enjoyed by non-experts, even children.

Eugenia Cheng on the Stephen Colbert show:
https://www.youtube.com/watch?v=mA402F5K47o

From the New York Times:
https://www.nytimes.com/2016/05/03/science/eugenia-cheng-math-how-to-bake-pi.html

General audience Books: How to bake $\pi$.
Beyond infinity: an expedition to the outer limits of mathematics

There will be further Special talks and events later.