ABSTRACTS: 2015 SUMMER VIGRE REU

All concepts mentioned in the abstracts will be carefully defined

APPRENTICE PROGRAM, weeks 1–5

Laci Babai and Madhur Tulsiani

Linear algebra and discrete structures

This is the same abstract as last year, but the material will not be the same! The course will develop the usual topics of linear algebra and illustrate them on (often striking) applications to discrete structures. Emphasis will be on creative problem solving and discovery. The basic topics include determinants, linear transformations, the characteristic polynomial, Euclidean spaces, orthogonalization, the Spectral Theorem, Singular Value Decomposition. Application areas to be highlighted include spectral graph theory (expansion, quasirandom graphs, Shannon capacity), random walks, clustering high-dimensional data, extremal set theory, and more.

FULL PROGRAM, weeks 1–8, ALL WELCOME

Probability

Antonio Auffinger, weeks 1-2

Random graphs and networks

This course will be an introduction to theory of random graphs. This theory began in the late 50s in several papers by Erdos and Renyi. It became incredibly popular in the late twentieth century with the notion of six degrees of separation and with study of human social networks and the Internet. The purpose of this course is to use a wide variety of mathematical arguments to obtain insights into the properties of these graphs, including their geometric and dynamical attributes.

Logic

D. Hirschfeldt (week 1) and M. Malliaris (week 2)

Keisler's order

Ultraproducts and ultrapowers give a way of taking limits of infinite sequences of mathematical objects, under very few conditions. We will explain this construction and some of its properties. We will then explain how it can be used to give a measure of relative complexity, and to define a large scale classification program in model theory (with many interesting open problems). Prior experience with logic may be useful but is certainly not required.

Dynamics

Kathryn Lindsey, weeks 1-2

Introduction to ergodic theory

Consider the sequence

$$p, f(p), f^2(p), f^3(p), ..., f^n(p)$$

for some function f from a space X to itself and a point p in X.

You could think of this sequence as describing how the state of some system changes over time, with each application of the function f corresponding to one unit of time. What happens as n goes to infinity? Will this sequence "spread out over the whole space X, or will it "get stuck in some small part of the space? Will the sequence spend "the same amount of time in all parts of the space? Will it ever return to where it started? How do the answers to these questions depend on our choice of starting point p, function f or space X? These questions, and the mathematical tools to rigorously ask and answer them, form the core of ergodic theory.

I will present an introduction to some basic concepts in ergodic theory. Specifically, I will discuss recurrence, ergodicity, the ergodic theorem, and mixing. In order to talk about these notions, I will first develop some concepts in measure theory.

Geometry

Jesse Wolfson, weeks 3-4

Algebraic functions and their surfaces

An algebraic function can be thought of as a map. In these lectures, we'll learn to read these maps using geometric ideas dating back to Riemann. Beginning with maps of the plane, we'll zoom out to study maps of the whole globe, and then we will use algebraic functions to discover and explore surfaces of less familiar worlds. In more formal (but less instructive) language, we'll get an introduction to complex functions in one variable, to "multi-valued functions" and Riemann surfaces, and, time permitting, to Riemann's original approach to the moduli space of algebraic curves. Key questions include: what kind of map is an algebraic function? How can you discover the shape of your world if you only have flat maps? How big is the space of all possible worlds?

Danny Calegari, week 6

An introduction to hyperbolic geometry and Kleinian groups, with a focus on concrete examples, and stressing connections to geometry, dynamics, topology and number theory.

Ian Le, week 6

The classical Lie groups, the orthogonal, unitary, and symplectic groups, are central objects in all of mathematics (and physics too). Their study draws together algebra, topology and geometry in a beautiful way. I will start with some of the basic theory of quadratic forms, which we will use to construct the classical Lie groups. Then we'll look at some spaces that these groups act on, like spheres and projective spaces. This will give us a feel for what the elements of these groups really "look like."

Howard Masur, week 7

Title: An introduction to the mathematics of billiards in polygons and straight lines on surfaces

Abstract: An appealing mathematical subject is to study the motion of a billiard ball in a polygon in the plane. We will start with the example of billiards in a square, where the behavior has been known for over 100 years. Then we will look at billiards on more exotic tables such as certain isoceles triangles. It turns out that in studying billiards one is led to study the geometry of surfaces that look flat. This set of three lectures will be devoted to an introduction to this subject.

Number Theory

Matthew Emerton, Keerthi Madapusi, Brandon Levin, weeks 1-4

Details will come soon

Analysis

Baoping Liu and Christophe Prange, weeks 1–2

Introduction to PDEs

These six lectures are an introduction to Partial Differential Equations, PDEs in short. Each lecture will cover one typical behavior, or one type of equation. We will see diffusive phenomena (heat, conduction of electricity, fluids) and dispersive effects (water waves, quantum mechanics). The main goal of the course is to show you the diversity of the field of PDEs and the connections to different areas of mathematics. We will emphasize some motivations, mathematical goals and dynamical aspects of the study of PDEs.

Applied Mathematics

Norman Lebovitz weeks 3–4

Periodic orbits of differential equations

The simplest solutions of ordinary differential equations are equilibrium points, and the next simplest are periodic orbits. Equilibrium points are too simple, so we consider the discovery of periodic orbits. For differential equations in the plane, i.e., two-dimensional systems, there is a general, geometric picture, the Poincarè-Bendixson theory. For higher-dimensional systems we'll need to sacrifice some generality, but will continue to find that geometric methods play a major role.

Aditya Khanna week 3

Some applications of mathematics in medicine

First talk: Networks

Over the past few years, online social networks have occupied a central position in our daily lives. But the study of networks, and indeed, the engagement within networks, is not new. Connections between entities, and the forces that drive those connections have provided fascinating avenues for reseach in a variety of disciplines, medicine, biology, ecology, and applied social and physical sciences. I will cover some of the mathematical ideas in the study of networks, and discuss how these ideas are applied to answer broader scientific questions.

Second talk: Epidemic Modeling

The use of visual and mathematical cues from data have provided a rich source of solving difficult problems in biology and medicine. IN this talk, I will focus on HIV/AIDS, the use of mathematical and statistical modeling to uncover transmission patterns of the virus, and discuss ongoing applications to design interventions to eliminate new HIV infections. This talk will build on some of the concepts we discuss in the first lecture, since network models provide a rich set of tools to conceptualize and compute the various biological and social interactions that drive HIV transmission.

Algebraic Topology

Peter May, Agnes Beaudry, Inna Zakharevich, Marc Stephan, weeks 1-8

Finite spaces and larger contexts

This is the same abstract as last year, but the material will not be the same! There is a fascinating and little known theory of finite topological spaces. We will present lots of the basic theory and how it relates to partially ordered sets (posets), classical simplicial complexes, finite groups, and categories.

For example, we shall show how to describe a space with $2n + 2$ points that to the eyes of algebraic topology is "just the same" as the n-sphere S^n. For another example, we shall reinterpret an interesting unsolved problem in finite group theory in terms of finite topological spaces. We shall go through a proof due to past REU participants of an unexpected result that they themselves discovered, and we shall give quite a few problems and questions. We will go slowly enough that all can follow (promise!!!), and we will introduce some more classical material to give context. However, most of the material is sure to be new to even the most advanced students.

A book in progress is available on line:
http://math.uchicago.edu/ may/FINITE/FINITEBOOK/FiniteAugBOOK.pdf
(You are offered a $1.00 reward for each typo you find.)