

ABSTRACTS: 2012 SUMMER VIGRE REU

All concepts mentioned in the abstracts will be carefully defined

APPRENTICE PROGRAM

LACI BABAI

Linear algebra and combinatorics

The course will develop the usual topics of linear algebra and illustrate them on highly unusual and often striking applications to combinatorics, discrete geometry, and discrete probability. Emphasis will be on creative problem solving and discovery.

The basic topics include permutations, determinants, linear transformations, the characteristic polynomial, Euclidean spaces, orthogonalization, the Spectral Theorem, linear algebra over finite fields. Application areas to be highlighted include, time permitting, extremal set theory, the spectral theory of graphs (expansion, mixing of random walks, independence number, Shannon capacity, etc.), k-wise independence of events, counting zero-patterns of polynomial maps, and more.

FULL PROGRAM

DAVID CONSTANTINE and WILLIAM LOPES

Title: Lie groups in geometry OR Lies, damned Lies and geometry

Description: We will give an elementary introduction to Lie groups and Lie algebras, followed by applications to geometry. We will work with matrix Lie groups, so no special knowledge of manifolds will be required, just a willingness to multiply matrices. We'll cover the basic definitions of Lie groups and Lie algebras and how they relate to one another, and we will see several examples of how they can be used to describe and study important geometric objects.

ANNE SHIU and EVA MARIE STRAWBRIDGEma

Mathematical Biology Short Courses

The math biology short courses will be partitioned into complementary but non-overlapping parts:

- 1) Mathematical Modeling and
- 2) Algebraic Statistics and Computational Biology.

Each course is stand-alone, meaning, participation in one does not require background from the other.

Mathematical modeling is the use of symbolic reasoning to address concrete questions or explain observable phenomena. The evolution of genetics, the spread of invasive species, or the effect of an irregular heart beat on the circulation of blood in the human body are all instances to which mathematical reasoning can be applied. Rather than delving into a particular field of mathematics, we will discuss

a series of topics including biogeography, transport of blood and oxygen, and muscle mechanics. All topics may be initially investigated with elementary mathematical techniques (e.g. calculus) but are also current fields of study.

Algebraic statistics employs techniques from abstract algebra for statistical inference. This course gives an introduction to algebraic statistics and its application in computational biology. Topics will include log-linear statistical models, expectation maximization, DNA sequence analysis, and phylogenetics. We will explain the mathematical connections to ideals, varieties, and tropical geometry. No prior knowledge of abstract algebra, statistics, or biology will be assumed.

MATT EMERTON, EDDIE HERMAN, and LIANG XIAO

Number Theory: Primes in Arithmetic Progressions and The Class Number Formula

Abstract: Dirichlet proved in the mid nineteenth century that there are infinitely many primes of the form $a+bn$ with fixed coprime numbers a and b . We aim to prove this result. This will require results from both algebra (ideal class groups) and analysis (generalizations of the Riemann zeta function). The class number formula ties these ideas together. If time permits, we will talk about the Birch Swinnerton-Dyer conjecture as an analog of the class number formula.

ANTONIO AUFFINGER and JONATHAN WEARE

Topics in Probability Theory

This course is divided into two parts.

The purpose of the first part is twofold. First, we will prove the main basic theorems in Probability Theory: the law of large numbers and the central limit theorem. Second, we will explore a few examples of active research topics. We will start with random walks, random graphs and geometries and end up with the statistics of eigenvalues of random matrices and a connection to the zeros of the Riemann Zeta function.

In the second part we will explore issues in Monte Carlo simulation. With applications ranging from molecular dynamics to statistical parameter estimation, Monte Carlo is one of the most important tools in modern scientific computing. We will introduce some of the basic tools like Markov chain Monte Carlo and importance sampling as well as explore theoretical issues related to their performance.

No background is necessary. There will be homework assignment to complement the lectures.

PETER MAY and ANGELICA OSORNO

There is a fascinating and little known theory of finite topological spaces. We will present lots of the basic theory and how it relates to partially ordered sets (posets), classical simplicial complexes, finite groups, and categories. For example, we shall show how to describe a space with $2n + 2$ points that to the eyes of algebraic topology is “just the same” as the n -sphere S^n . For another example, we shall reinterpret an interesting unsolved problem in finite group theory in terms of finite topological spaces. We shall go through a proof due to past REU participants of an unexpected result that they themselves discovered, and we shall give quite a few problems and questions. We will go slowly enough that all can follow (promise!!!), but the material is sure to be new to even the most advanced students.