PROJECT SUMMARY

The PI proposes to work under the sponsorship of Hal Sadofsky at the University of Oregon. The PI has developed a bicategorical perspective on Morita theory in classical and modern (derived) algebra and topology. Furthermore, the PI has given a characterization of generalized Azumaya algebras in categorical settings which provides a link between conceptual and calculational work. Classically, this link is essential to the theory of Brauer groups, and the proposed work will study consequences of this characterization in topological settings.

Objectives and Methods. The main objective of the proposed work is to develop a calculational approach to Brauer groups of ring spectra. The PI will use a characterization of Azumaya objects to make more detailed study of a categorical exact sequence, developed by Vitale [Vit02], which relates categorical Picard and Brauer constructions. The experience of Hal Sadofsky with Picard group calculations for spectra will be an important resource for this project.

A secondary project was initiated by observations of Peter May relating current work in algebraic geometry to the categorical exact sequence developed by Vitale. This work would realize a spectrum-level version of this sequence, shedding light on both the algebraic topology and the algebraic geometry. Work of Duskin [Dus] develops a similar realization in simplicial sets, and would be a starting point for this project.

In joint work, Justin Noel and the PI are developing calculational evidence for H_∞ structure on the Brown-Peterson spectrum, BP. The calculations make use of classical reductions to power-series formulas, and then computer approximations to these power series. The proposed project would continue this work, with the goal of re-energizing interest in basic questions about the structure of BP which have remained unanswered for more than 30 years.

Intellectual Merit. The proposed research is part of a general interest in developing algebraic tools in topological and other generalized contexts. Much of the classical calculations of Brauer groups are carried out by ideal-theoretic arguments. However, there is no understood ideal theory for ring spectra, and hence alternative approaches are needed. The proposed research would develop such alternative approaches, with an eye toward general results which would be applicable in algebro-geometric and other homotopy-theoretic contexts.

The question of whether Vitale's sequence can be realized topologically will shed light on general categorical questions about the relationship between weak n-groupoids and homotopy n-types, since the realizing spectra would have at most three non-trivial homotopy groups.

Broader Impact. The PI's proposed work at the University of Oregon will take place in a community of research and education. The PI has had experience as a graduate student mentor in the University of Chicago's Directed Reading Program and Research Experience for Undergraduates program, and will continue to contribute to mentorship programs in the University of Oregon community.

The categorical content of the proposed work serves to enhance infrastructure for research on Picard and Brauer groups throughout the mathematical disciplines. Morita theory has been developed in a broad range of mathematical disciplines, and one difficulty in this subject is that the various disciplines have developed different language and different foci; a categorical perspective can foster unified language and facilitate translation. This, in turn, improves communication and transfer of ideas across disciplines.

The calculational work with BP has the potential to appeal to an audience with minimal mathematical background. The reduction from a subtle topological question to a series of combinatorial problems puts this structural question in a form that can be easily communicated. This has the potential to generate broader interest from the public and participation from undergraduate math students.

The PI has also made contributions to the University of Chicago's Science in Art Exhibit. Support from the MSPRF will enable the PI to continue participation in a community of scientists interacting with the broader public through visual arts. This interaction supports public awareness of and interest in science, and develops scientifically focused interactions between scientists and non-scientists.

References

[Dus] JW Duskin, *The Azumaya complex of a commutative ring*, Lecture Notes in Math **1348**, 107–117.

[Vit02] E.M. Vitale, *A Picard–Brauer exact sequence of categorical groups*, Journal of Pure and Applied Algebra **175** (2002), no. 1-3, 383–408.