1. Sketch of how to think equivariantly

Let $\mathscr{V}$ be as usual, $G\mathscr{V}$ the category of G-objects in $\mathscr{V}$ with its induced symmetric monoidal structure, denoting $\mathscr{V}_G(X,Y)$ the object in $G\mathscr{V}$ given by $\mathscr{V}(X,Y)$ with its conjugation action by the discrete group G. This will be a formal toy model. We will later want $\mathscr{V}$ to be a good category of spectra, $G\mathscr{V}$ to be a good category of G-spectra enriched over $\mathscr{V}$ (and not literally of the form $G\mathscr{V}$).

We can apply our general theory to $G\mathscr{V}$, viewed as enriched over $\mathscr{V}$ via the hom objects $\mathscr{V}_G(X,Y)^G$ in $\mathscr{V}$. In fact, we can start over with any $\mathscr{V}$-enriched $\mathscr{M}$ and form $G\mathscr{M}$ and $\mathscr{M}_G$ analogously, enriched over $\mathscr{V}$ and $G\mathscr{V}$ respectively. Let $G\mathscr{D}[full]$ be an appropriate full subcategory of $G\mathscr{M}$, enriched over $\mathscr{V}$ via the $\mathscr{M}_G(X,Y)^G$ and let $\mathscr{D}_G[full]$ denote the corresponding full subcategory of $\mathscr{M}_G$, enriched over $G\mathscr{V}$ via the hom objects $\mathscr{M}_G(X,Y)$. We give meaning to the following diagrammatic road map, which is written on the level of ordinary model categories.

$$\begin{array}{ccccc} G\mathscr{M}_0 & \underset{\mathbb{U}}{\overset{\mathbb{T}}{\rightleftarrows}} & \mathscr{P}(G\mathscr{D}[full],\mathscr{V})_0 & \underset{(-)^G}{\overset{\varepsilon^*}{\rightleftarrows}} & \mathscr{P}_G(\mathscr{D}_G[full],G\mathscr{V})_0 \\ &&&& \delta^* \downarrow \;\uparrow \delta^{\#} \\ \mathscr{P}(G\mathscr{D}[disc],\mathscr{V}_0) & \underset{(-)^G}{\overset{\varepsilon^*}{\rightleftarrows}} & \mathscr{P}_G(\mathscr{D}_G[disc],G\mathscr{V}_0) & \overset{\cong}{\longleftarrow} & \mathscr{P}_G(S_G[\mathscr{D}_G[disc]],G\mathscr{V})_0 \end{array}$$

Step 1.1. *Prove an enriched Quillen equivalence between $G\mathscr{M}$ and $\mathscr{P}(G\mathscr{D}[full],\mathscr{V})$.*

This we already know how to do: our general theory applies.

Step 1.2. *Prove a Quillen equivalence (probably enriched over $\mathscr{V}$ but that's gravy) between the underlying categories of $\mathscr{P}(G\mathscr{D}[full],\mathscr{V})$ and $\mathscr{P}_G(\mathscr{D}_G[full],G\mathscr{V})$.*

If $\varepsilon\colon G \longrightarrow e$, then ε^* is pullback and assigns trivial action of G to objects of $\mathscr{V}$. Its right adjoint is $(-)^G$. We use $(-)^G$ to create a model structure on $\mathscr{P}_G(\mathscr{D}_G[full],G\mathscr{V})_0$. It perhaps requires special good properties of the fixed point functor, but this too we know how to do in principle. It really depends only on $\mathscr{V}$, not $\mathscr{M}$: we could start with any pair $(\mathscr{D}_G, G\mathscr{D})$. Since $\eta\colon X \longrightarrow (\varepsilon^* X)^G$ is the identity in our current formal context, this will certainly be a Quillen equivalence. NO: ε^* is not defined on presheaves! However, $(-)^G$ is fine. Alternative prolongation construction?

Step 1.3. *Find a discrete G-category $\mathscr{D}_G[disc]$ with G-fixed point category $G\mathscr{D}[disc]$ together with a G-equivalence $\mathscr{D}_G[disc] \longrightarrow \mathscr{D}_G[full]$. Then use it to construct an enriched Quillen equivalence between $\mathscr{P}_G(\mathscr{D}_G[full],G\mathscr{V})$ and $\mathscr{P}_G(S_G[\mathscr{D}[disc]],G\mathscr{V})$.*

The first part should be the hard part, and in the G-spectrum case it is the thing that we have been looking for in terms of transfers on the one hand and our understanding of $F_G(X,Y)$ for orbit G-spectra X and Y, or really, for any G-spectra $\Sigma_G\Sigma^\infty(K)_+$, where K is a G-set, on the other hand. Obviously, in general the answer depends on the initial choice of $\mathscr{M}$. In our G-spectrum case, it is "only" a case of finding a good description of $(\mathscr{D}_G, G\mathscr{D})$ and a map δ. We know that it is there morally from our understanding of the G-homotopy types of the $F_G(\bar{K},\bar{L})$, where $\bar{K} = \Sigma_G^\infty(K_+) = S_G[K_+]$ for a finite G-set K: they are $S_G[(K\times L)_+]$.

Step 1.4. *Prove that $\mathscr{P}_G(S_G[\mathscr{D}_G[disc]],G\mathscr{V})_0$ is isomorphic as a model category to $\mathscr{P}_G(\mathscr{D}_G[disc],G\mathscr{V}_0)$.*

This should be the same as we saw in general, in the nonequivariant context. The notation $\mathscr{P}_G$ is really more for emphasis than for mathematics. Again, this should be a general argument, independent of $\mathscr{M}$.

Step 1.5. *Prove a Quillen equivalence (no enrichment) between* $\mathscr{P}_G(\mathscr{D}_G[disc], G\mathscr{V}_0)$ *and* $\mathscr{P}(G\mathscr{D}[disc], \mathscr{V}_0)$.

This should be essentially the same as Step 1.2, but perhaps simpler, and again general, independent of $\mathscr{M}$. As a test of sanity, we should make sure that this program works trivially when $\mathscr{M} = \mathscr{V} = \mathscr{U}$ and $G\mathscr{D} = G\mathscr{O}$; $\mathscr{O}_G$ will have orbits and set maps between them, with G acting by conjugation.