The proof that the equivariant linear isometries operad is an E_∞ G-operad is parallel to the nonequivariant proof, but you are right that I seem never to have written it up. It belongs in the infinite loop space paper. Let Λ be a subgroup of $G \times \Sigma_n$. Since Σ_n acts freely, there can be no Λ-fixed point if $\Lambda \cap \Sigma_n$ has a point other than e. Let $\Lambda \cap \Sigma_n$ be trivial. Then there is a subgroup H of G and a homomorphism $\rho\colon H \longrightarrow \Sigma_n$ such that Λ is the set of points $(h, \rho(h))$. We may view $\mathscr{L}(n)$ as an H-space by pullback along this isomorphism from H to Λ. So it suffices to prove that $\mathscr{L}(n)$ is H-contractible under this pullback action. At this point, Σ_n is irrelevant, and we can replace G by H and consider a finite or countable dimensional G-representation V, a complete G-universe U, and the G-space $E = \mathscr{I}(V, U)$. It suffices to prove that E^G is contractible, and it is certainly non-empty since V is G-isomorphic to a subrepresentation of U.

We mimic the proof of I.1.3 in E_∞ ring spaces and E_∞ ring spectra. Write U as the direct sum $\sum_\rho V_\rho^\infty$, where the V_ρ run through the irreducible representations ρ of G. We divide V_ρ^∞ as the direct sum of the even and the odd numbered copies of V_ρ. Define $\alpha\colon U \longrightarrow U$ to be the obvious G-linear isometry mapping the i^{th} copy of each V_ρ to the $2i^{\text{th}}$ copy. Define a G-linear isomorphism $\beta\colon U \longrightarrow U \oplus U$ by mapping the $2i^{\text{th}}$ copy of V_ρ to the i^{th} copy in the first summand and the $(2i-1)^{\text{st}}$ copy to the i^{th} copy in the second summand. Then $\beta \circ \alpha = i_1$, the inclusion of the first summand. There is a path H_1 in $\mathscr{I}(U, U)^G$ from the identity to α and a path H_2 in $\mathscr{I}(V, V \oplus V)^G$ from i_1 to i_2. They are obtained by normalizing the G-linear paths J_1 and J_2 given by

$$J_1(t)(v) = (1-t)v + t\alpha(v) \quad \text{and} \quad J_2(v) = (1-t)i_1(v) + ti_2(v)$$

For each t, these are G-linear inclusions; that is $J_1(t)(v) = 0$ implies $v = 0$ and similarly for J_2. Choose any fixed $\gamma \in \mathscr{I}(V, U)^G$ and define a homotopy

$$H\colon I \times \mathscr{I}(V, U)^G \longrightarrow \mathscr{I}(V, U)^G$$

by

$$H(t, f) = \begin{cases} H_1(2t) \circ f & \text{if } 0 \le t \le 1/2 \\ \beta^{-1} \circ (f \oplus \gamma) \circ H_2(2t-1) & \text{if } 1/2 \le t \le 1. \end{cases}$$

Then $H(0, f) = f$ and $H(1, f) = \beta^{-1} i_2 \gamma$, which is independent of f.