

BASIC ALGEBRA CHECKLIST

(If you can do all these problems, you don't need to go through problem sheets 1-6)

- 0.1.** Prove that $k \in \mathbb{Z}_n$ is a generator if and only if $\gcd(k, n) = 1$.
- 0.2.** Prove that in any group the orders of elements ab and ba are equal.
- 0.3.** Prove that the following three definitions of a normal subgroup are equivalent:
- (1) $H \subset G$ is a normal subgroup if for all $g \in G$ $gHg^{-1} = H$.
 - (2) $H \subset G$ is a normal subgroup if left and right cosets of H in G coincide.
 - (3) $H \subset G$ is a normal subgroup if there is a homomorphism $f : G \rightarrow G'$ to some group G' , with kernel H .
- 0.4.** Let G be the group of symmetries of the square, and let N be its subgroup generated by the central symmetry. Prove that N is normal and find the quotient group.
- 0.5.** Find the conjugacy classes in the group of symmetries of the square.
- 0.6.** Prove that the commutator subgroup is normal.
- 0.7.** Find the parity of a cycle of order n (determine whether it is even or odd).
- 0.8.** Find the conjugacy classes in A_4 and A_5 .
- 0.9.** Prove that S_4 is solvable, and S_5 is not.
- 0.10.** Prove that for $0 \neq z \in \mathbb{C}$ the function $\sqrt[n]{z}$ takes exactly n values.