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APPROXIMATE MARKED LENGTH SPECTRUM RIGIDITY

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(M, g) negatively curved surface

- **Marked length spectrum** (MLS_g)

  function that keeps track of lengths of closed geodesics

- **Rigidity** (Otal '90, Croke '91)

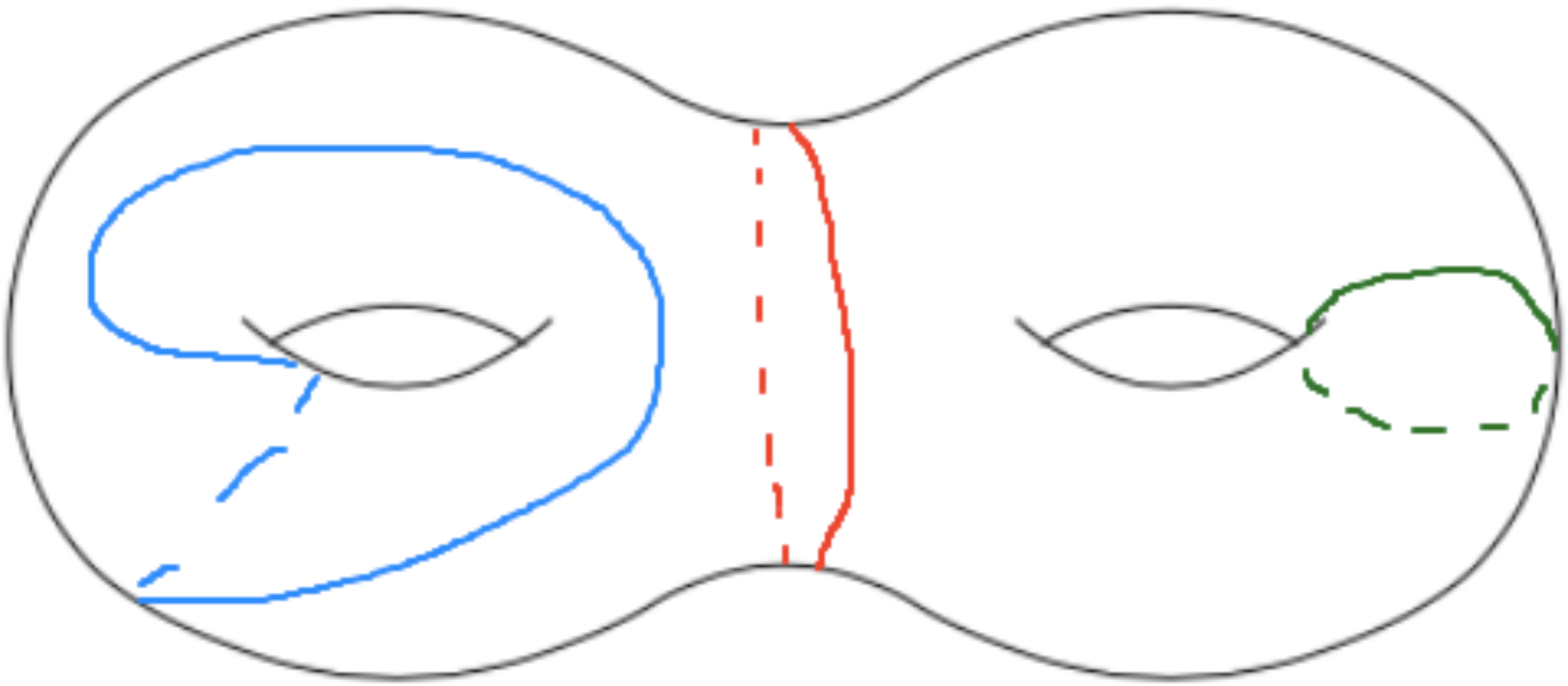
  $\mathrm{MLS}_{g_1} = \mathrm{MLS}_{g_2} \implies g_1 = g_2$

- **Approximate rigidity** (B '20)

  $\mathrm{MLS}_{g_1} \approx \mathrm{MLS}_{g_2} \implies g_1 \approx g_2$

THE LENGTH SPECTRUM

Definition: The length spectrum of *(M, g)* is the set of lengths of all closed geodesics



THE LENGTH SPECTRUM

What does the length spectrum tell us about the underlying metric?

For closed hyperbolic surfaces,

length spectrum $\longleftrightarrow$ Laplace spectrum

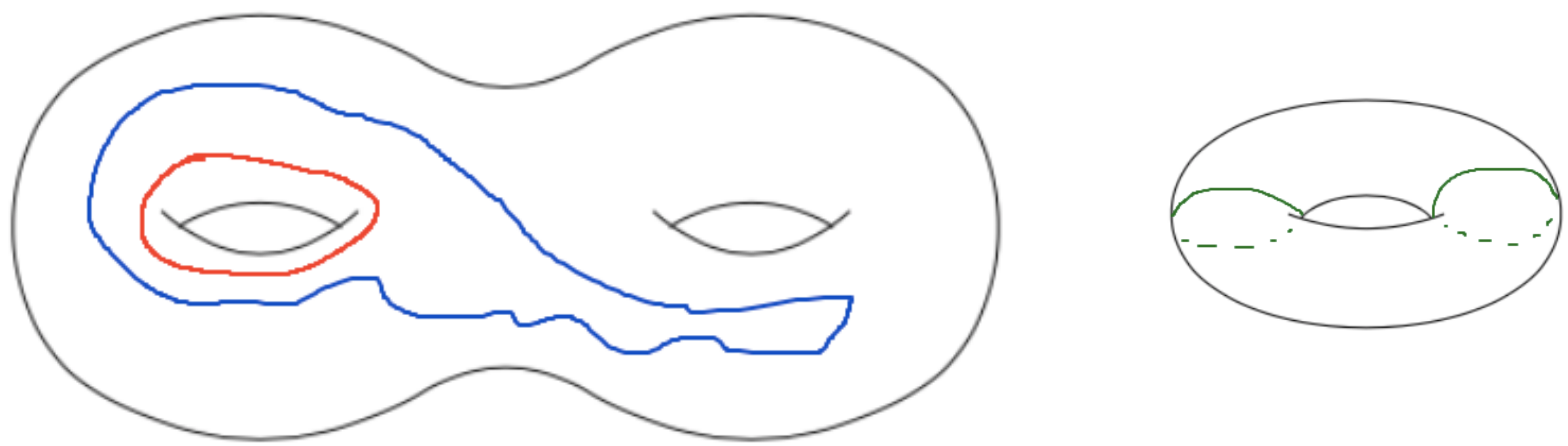
(Huber '59)

"Can you hear the shape of a drum?"

No (Vignéras '80)

THE MARKED LENGTH SPECTRUM

Fact: Every closed curve in M is freely homotopic to a unique closed geodesic



$$\mathrm{MLS}_g : \left\{ \begin{matrix} \text{free homotopy} \\ \text{classes} \end{matrix} \right\} \longrightarrow \mathbb{R}$$

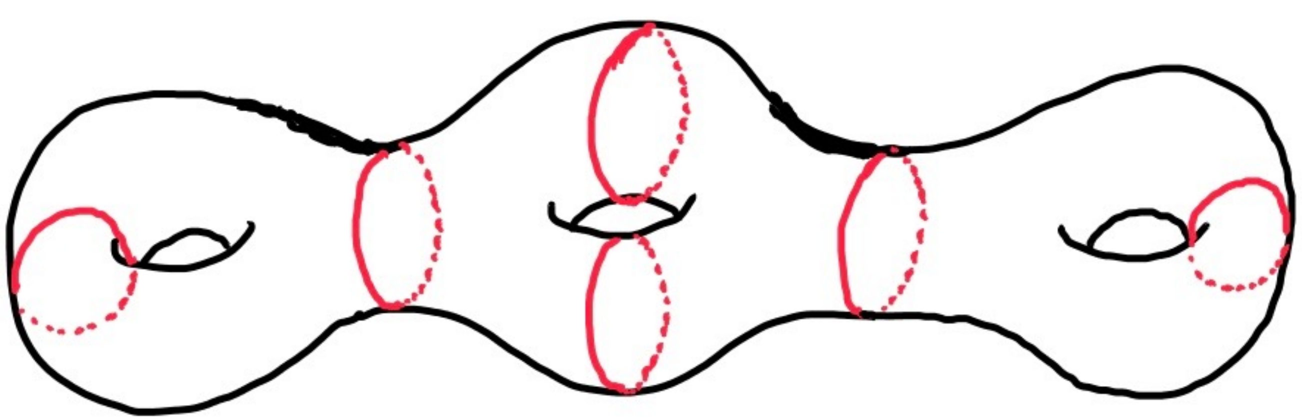
$$\begin{matrix} \text{free homotopy} \\ \text{class} \end{matrix} \longmapsto \begin{matrix} \text{length of closed} \\ \text{geodesic} \end{matrix}$$

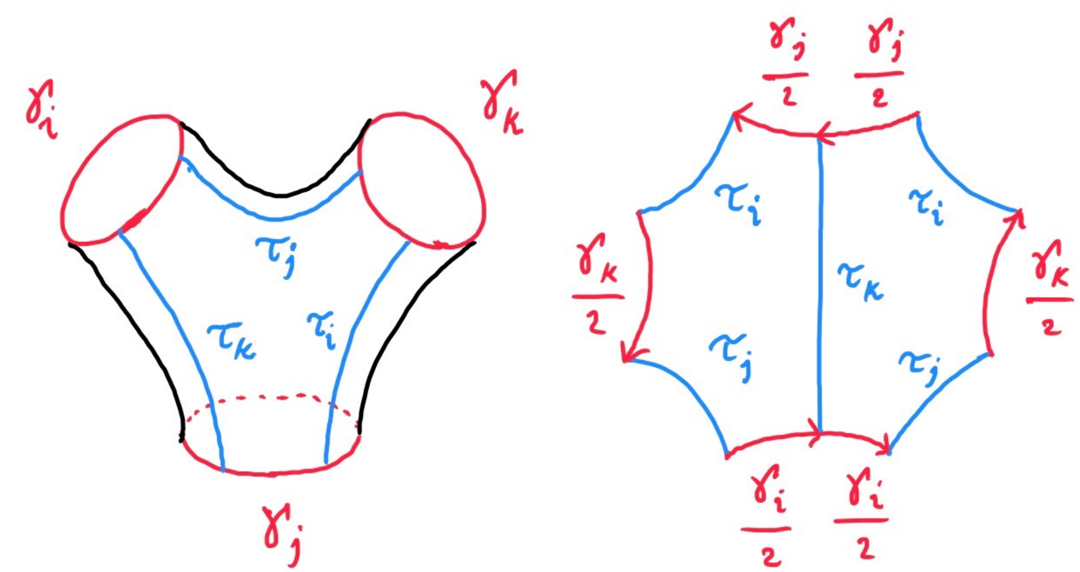
What does the marked length spectrum tell us about the underlying metric?

9g-9 theorem

For any hyperbolic surface of genus *g*, there are *9g-9* simple closed curves whose lengths determine the surface up to isometry

$\Longrightarrow$ MLS rigidity





APPROXIMATE RIGIDITY - CONSTANT NEGATIVE CURVATURE

Theorem (Thurston '98): For hyperbolic surfaces,

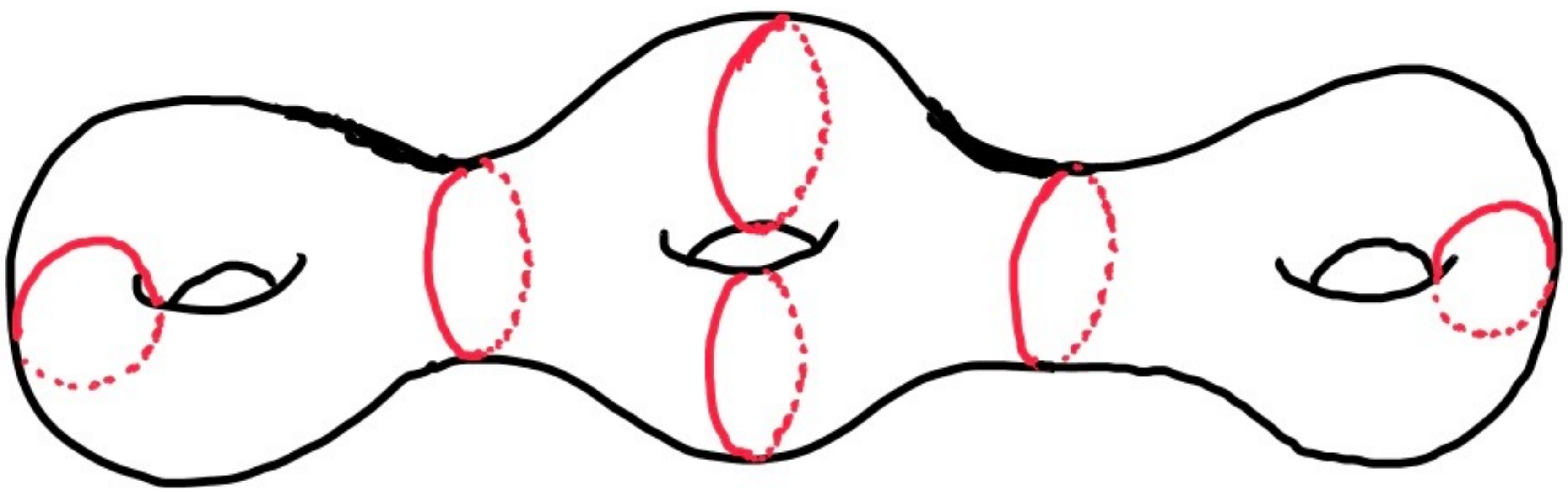
$$\mathrm{MLS}_{g_1} \approx \mathrm{MLS}_{g_2} \implies g_1 \approx g_2$$

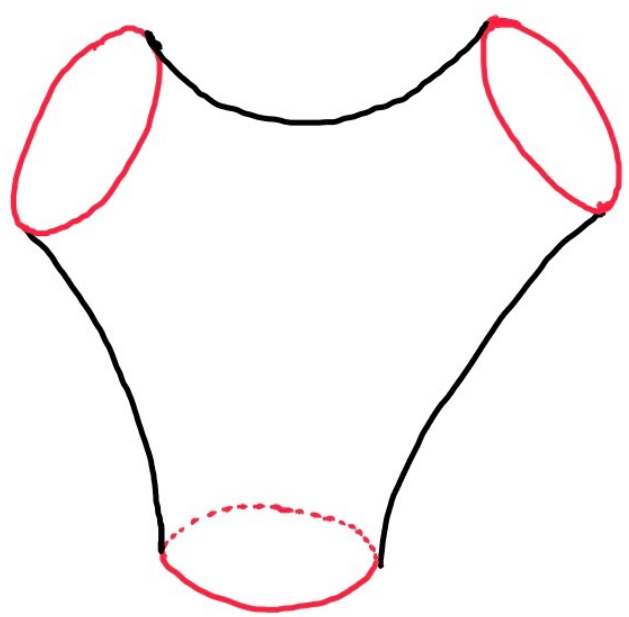
maximal length ratio = minimal Lipschitz constant

VARIABLE NEGATIVE CURVATURE

Analogy

constant curvature	$\longleftrightarrow$	variable curvature
holomorphic functions		smooth functions





MLS RIGIDITY IN VARIABLE NEGATIVE CURVATURE

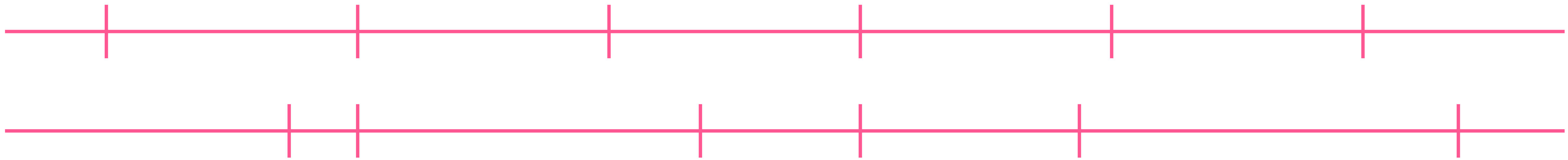
Theorem (Otal '90, Croke '91)

A negatively curved metric on a closed surface is determined up to isometry by its marked length spectrum

$$\mathrm{MLS}_{g_1} = \mathrm{MLS}_{g_2} \implies g_1 = g_2$$

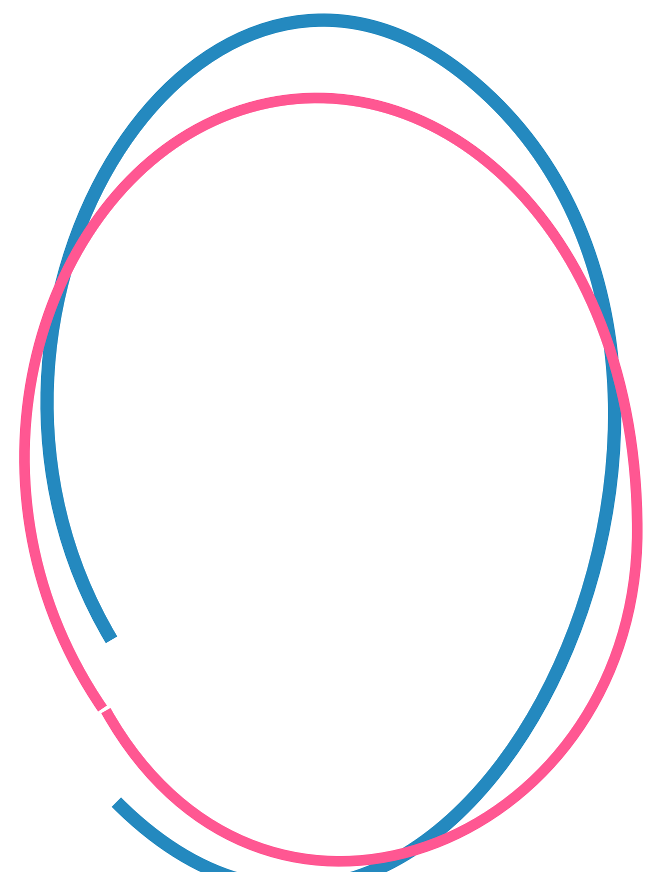
METHODS - DYNAMICAL SYSTEMS

Part 1: construct a map that preserves geodesics



Part 2: construct a map that preserves distances

The geodesic flow $\varphi^t : T^1M \to T^1M$ is Anosov



Anosov closing lemma: "almost closed orbits" are shadowed by closed orbits

APPROXIMATE RIGIDITY

Main question:

$$\mathrm{MLS}_{g_\varepsilon} \approx \mathrm{MLS}_g \implies g_\varepsilon \approx g$$

$$1 - \varepsilon \leq \frac{\mathrm{MLS}_{g_\varepsilon}}{\mathrm{MLS}_g} \leq 1 + \varepsilon$$

APPROXIMATE MLS RIGIDITY IN PINCHED NEGATIVE CURVATURE

Theorem (B '20)

Let (M, g) and (M, g_ε) be metrics of pinched negative curvature satisfying

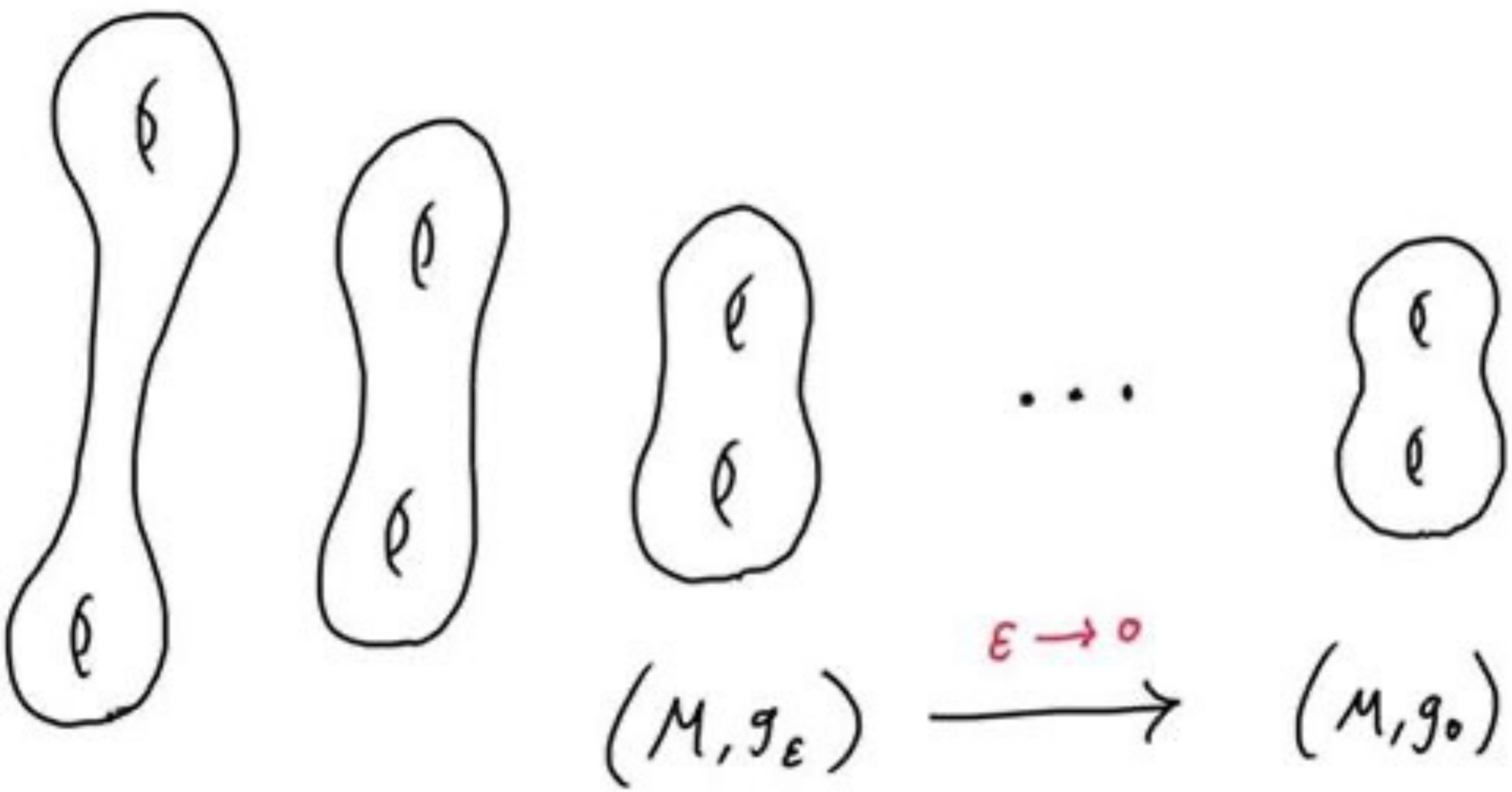
$$1 - \varepsilon \leq \frac{\mathrm{MLS}_{g_\varepsilon}}{\mathrm{MLS}_g} \leq 1 + \varepsilon$$

Then, given $A > 1$, there exists ε small enough such that there is a map $f : M \to M$ satisfying

$$\frac{1}{A} \leq \frac{d_{g_\varepsilon}(f(p), f(q))}{d_{g_0}(p, q)} \leq A$$

METHODS

Step 1: Gromov limit (Pugh '87)

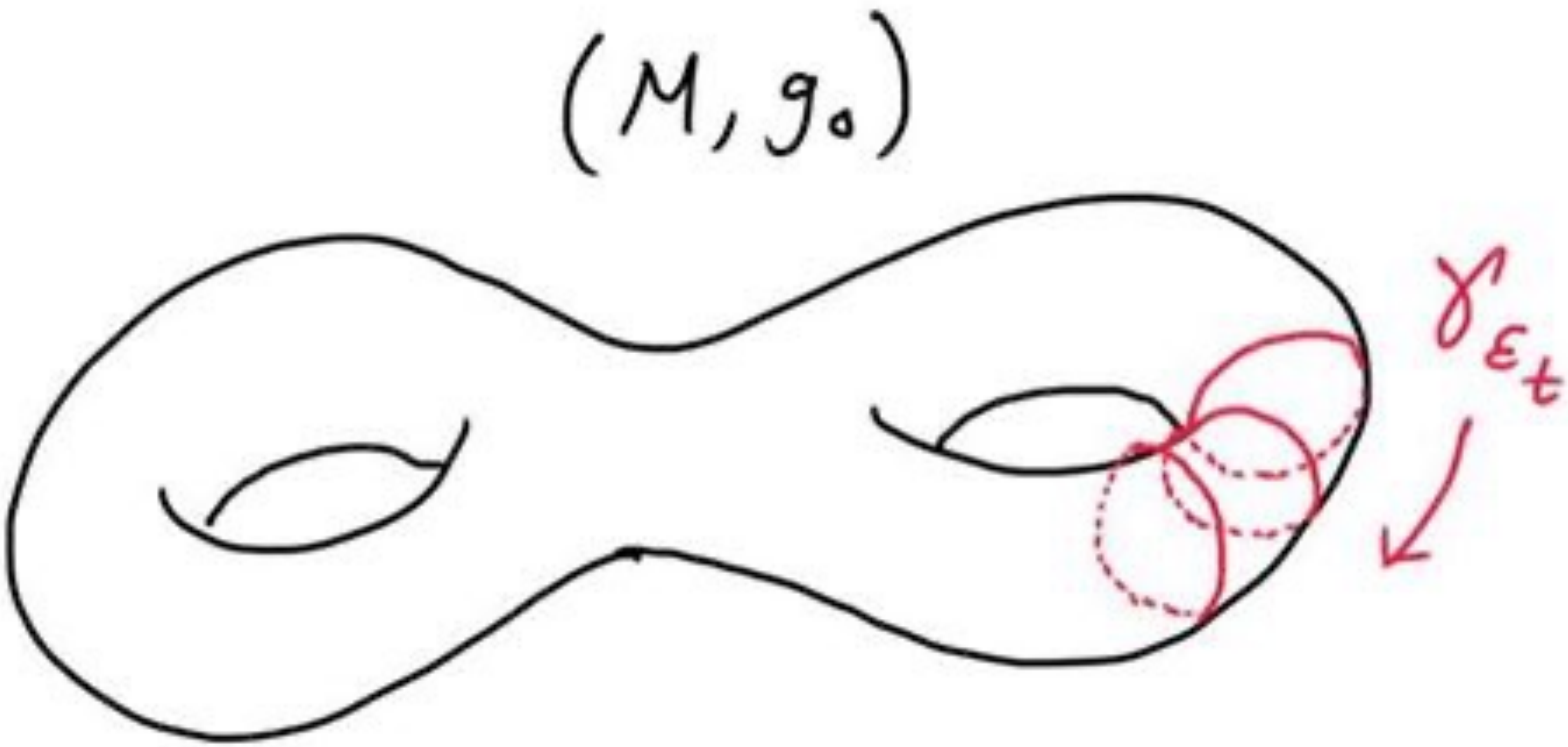


Lipschitz

$$\mathrm{MLS}_{g_0} = \mathrm{MLS}_g$$

METHODS

Step 2: Adapt Otal's proof to show (M, g_0) is isometric to (M, g)



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