Week 7, Due Fri 11/15

1. Prove — with full details using εs and δs — that

$$\lim_{x \to 1} x^2 = 1.$$

2. Write down a set of numbers A with all of the following properties, and justify your answer:

   (a) $\sup A$ exists and is not in A.

   (b) $\inf A$ exists and lies in A.

3. For each of the following conditions, determine whether or not such an $f(x)$ has to be continuous at $x = 0$ or not:

   (a) $\exists \varepsilon > 0, \forall \delta > 0,$ if x satisfies $0 < |x| < \delta,$ then $|f(x) - f(0)| < \varepsilon.$

   (b) $\forall \varepsilon > 0, \forall \delta > 0,$ if x satisfies $0 < |x| < \delta,$ then $|f(x) - f(0)| < \varepsilon.$

   (c) $\forall \varepsilon > 0, \exists \delta > 0,$ if x satisfies $0 < |x| < \delta,$ then $|f(x) - f(0)| < \varepsilon^2.$

4. Let $f(x)$ be a continuous function on $\mathbf{R}$ satisfying $f(x+1) = f(x)$ for every x. Prove that $f(x)$ is bounded on $\mathbf{R}$.