Week 5, Due Fri 11/1

=

1. Spivak, Chapter 5, Problem $10d, 32$.

2. For a real number x, let $\lfloor x \rfloor$ denote the largest integer n with $x \geq n$. Let

$$f(x) = \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor$$

   Prove that the following limits do not exist.

   (a) $\lim_{x \to 0} f(x)$
   (b) $\lim_{x \to 1} f(x)$

3. Suppose that $f(x)$ is an increasing function, which means that if $x > y$ then $f(x) > f(y)$. Suppose that $\lim_{x \to 0} f(x)$ exists. Prove that the limit is equal to $f(0)$.