Superrigidity and mapping class groups

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November 5, 1997

1 Introduction

Margulis's superrigidity theorem (see [Zi1]) says that if Γ is an irreducible lattice in a real semisimple[1] Lie group G of rank at least two, then any homomorphism $\Gamma \to H$ into a simple Lie group H has bounded image or extends to a homomorphism $G \to H$, and the image is always bounded when H is a p-adic group. In this note we give two examples of nonlinear superrigidity: one for mapping class groups and Teichmüller space, another for diffeomorphism groups of surfaces.

Let S be a compact, orientable, connected surface, possibly with boundary. The *mapping class group* $\mathrm{Mod}(S)$ is the group $\mathrm{Homeo}^+(S)/\mathrm{Homeo}_0(S)$ of isotopy classes of orientation-preserving homeomorphisms of S. The proof of our first result will come directly from theorems of Kazhdan-Margulis and Kaimanovich-Masur, together with some structure theory of $\mathrm{Mod}(S)$.

Theorem 1.1 (Superrigidity with $Mod(S)$ target) *Let Γ be an irreducible lattice in a semisimple lie group G of $\mathbf{R}$-rank at least two. Then any homomorphism $\phi : \Gamma \to Mod(S)$ has finite image.*

Remarks.

1. Theorem 1.1 was conjectured by Ivanov [Iv1, Iv2], and occurs as Problem 2.15 in Kirby's problems list [Ki]. Ivanov proved [Iv2] that ϕ cannot be an isomorphism, and in fact proved the stronger result that $\phi(\Gamma)$ cannot be a subnormal subgroup unless it is central.

[*]The first author is supported in part by NSF grant #DMS 9704640. The second author is supported in part by NSF grant #DMS 9201321.

[1]By *semisimple* we mean connected (in the real case) semisimple with finite center and no nontrivial compact factors.

2. Theorem 1.1 is false for $G = SO(n,1), SU(n,1)$ since, for example, there are lattices Γ in such G which admit surjective homomorphisms to $\mathbf{Z}$. We don't know if the theorem holds for lattices Γ in $SP(n,1)$ and F_4, although Corlette [Co] has proven the usual superrigidity for these lattices.

3. The action of $\mathrm{Mod}(\Sigma_g)$ on $H_1(\Sigma_g)$ preserves the natural intersection pairing on $H_1(\Sigma_g)$, and gives the well-known exact sequence

$$1 \to T_g \to \mathrm{Mod}(\Sigma_g) \to \mathrm{Sp}(2g, \mathbf{Z}) \to 1$$

where $\mathrm{Sp}(2g, \mathbf{Z})$ is the symplectic group and T_g is the Torelli group. As $\mathrm{Sp}(2g, \mathbf{Z})$ is an irreducible lattice in the semisimple Lie group $\mathrm{Sp}(2g, \mathbf{R}), g \geq 2$, Theorem 1.1 implies that this exact sequence doesn't split.

4. Theorem 1.1 holds with $\mathrm{Mod}(S)$ replaced by the braid group B_n. We prove this in §4.

In [FS2] a topological characterization of the action of $SL(3, \mathbf{Z})$ on the 3-torus among all lattice actions on compact 3-manifolds is given. The following restatement of Theorem 1.1 is a key ingredient in the proof of this result.

Theorem 1.2 (Surface actions are virtually isotopically trivial) *Let Γ be an irreducible lattice in a semisimple lie group G of $\mathbf{R}$-rank at least two. If Γ acts by homeomorphisms on the compact surface S, then there is a finite index subgroup $\Gamma' < \Gamma$ such that every $g \in \Gamma'$ acts by a homeomorphism isotopic to the identity. In particular Γ' acts homologically trivially on S.*

One simply lets Γ' be the kernel of the composition $\Gamma \to \mathrm{Homeo}^+(S) \to \mathrm{Homeo}^+(S)/\mathrm{Homeo}_0(S) = \mathrm{Mod}(S)$ and applies Theorem 1.1.

Let $\mathrm{Teich}(S)$ denote the Teichmüller space of the surface S. The isometry group[2] of $\mathrm{Teich}(S)$ is $\mathrm{Mod}(S)$ [Ro]. Kerckhoff [Ke] proved that any finite subgroup of $\mathrm{Mod}(S)$ can be realized as a group of isometries of some Riemann surface $X \in \mathrm{Teich}(S)$. Hence Theorem 1.1 gives:

Corollary 1.3 (Global fixed point) *Let Γ be an irreducible lattice in a semisimple Lie group G of $\mathbf{R}$-rank at least two. Then any isometric action of Γ on a Teichmüller space $\mathrm{Teich}(S)$ has a global fixed point.*

[2] in either the Teichmüller or Weil-Petersson metrics

The Zimmer program studies representations with target $H = \mathrm{Diff}(M)_\omega$; that is, actions of lattices and Lie groups on compact manifolds M preserving various geometric structures ω (see [Zi3] for a survey, and [Zi2] for the case of M a surface). Examples of superrigidity with target $\mathrm{Diff}(M)$ are [Wi, We, FS2]. The following theorem provides some more examples.

Theorem 1.4 ($SL(n, \mathbf{Z})$ actions on surfaces) *Let Σ_g denote the closed surface of genus g. Then for every $g \geq 2$ there exists $N > 0$ so that, for all $n \geq N$, every homomorphism $SL(n, \mathbf{Z}) \to Homeo^+(\Sigma_g)$ is trivial. More precisely, we can choose N to be any number satisfying*

$$2^{N(N-1)/2} \prod_{i=2}^{N-1} (2^i - 1) > 84(g-1)$$

Remarks.

1. In case the action preserves a connection and volume density, Theorem 1.4 follows from the results of [Zi4].
2. Nontrivial actions are not precluded *a priori* by algebra: $SL(n, \mathbf{Z})$ even acts faithfully by automorphisms on $H_1(\Sigma_g) \approx \mathbf{Z}^{2g}$ for $n \leq g/2$. One might guess (incorrectly) that some such action might come from an action on Σ_g.
3. The conclusion of Theorem 1.4 is false without some bound on N in terms of g: $SL(n, \mathbf{Z})$ maps onto $SL(n, \mathbf{Z}/2\mathbf{Z})$, and any finite group acts faithfully by automorphisms of some (necessarily high genus) Reimann surface (see Proposition 3.1 below).
4. Shmuel Weinberger has indicated (personal communication) an argument to show that an analogue of Theorem 1.4 with "finite image" replacing "trivial image" should hold for all compact manifolds, although in the case of surfaces the bound on N is worse than that given by Theorem 1.4.
5. It was recently shown in [FS1] that any real-analytic action of any finite index subgroup of $SL(n, \mathbf{Z}), n \geq 5$ on a compact, higher-genus surface must factor through a finite group action.

The key to our proof of Theorem 1.4 is, following Weinberger [We], to see how finite subgroups act.

Acknowledgements. We thank J. Alperin, R. Narasimhan, S. Smith, S. Weinberger, and R. Zimmer for many useful discussions. After we finished writing this paper we became aware that N. Ivanov had previously suggested an outline for Theorem 1.1 on his web page. He also has (personal communication) a different proof along lines that follow the approach of Margulis to superrigidity.

2 Proof of Theorem 1.1

Unless otherwise specified, by *surface* we will mean a compact, orientable, connected 2-manifold with (perhaps empty) boundary. For background, definitions, and references on $Mod(S)$ and Thurston's theory of surface diffeomorphisms we refer the reader to [Th, FLP, Iv3].

We emphasize that when S has boundary, an element of $\mathrm{Mod}(S)$ is an isotopy class of homeomorphisms which need not fix an invariant boundary component pointwise, and where the isotopy need not fix such a boundary component pointwise. Hence if S has boundary components $T_1, \ldots, T_r$ then $\mathrm{Mod}(S)$ is isomorphic to the mapping class group $\mathrm{Mod}(T)$ of the punctured surface T obtained by gluing a once-punctured disc D_i to each T_i along ∂D_i.

To prove the theorem we induct on $\tau(S) = 3g - 3 + n$, where g is the genus of S and n is the number of boundary components of S. We may also define $\tau(S)$ as the maximal number of isotopy classes of disjoint, simple, homotopically nontrivial, nonperipheral (i.e. not isotopic to ∂S), closed curves in S. In particular, if γ is a simple closed curve in S, and if R is the surface-with-boundary obtained from S by cutting along γ, then R is orientable and $\tau(R) < \tau(S)$.

To begin with, note that the theorem is true for all surfaces S with $\tau(S) \leq 0$: if S is an annulus, sphere, pair of pants, or disc then $Mod(S)$ is trivial; if S is a torus then $\mathrm{Mod}(S) \approx SL_2(\mathbf{Z})$ and the theorem follows from Margulis's superrigidity theorem (see [Zi1], Theorem 5.1.2).

Now let S be any surface with $\tau(S) > 0$, and suppose we know the theorem is true for all surfaces T with $\tau(T) < \tau(S)$. Let Γ be any irreducible lattice in a semisimple Lie group G of rank at least two, and let $\phi : \Gamma \to \mathrm{Mod}(S)$. We suppose that $\phi(\Gamma)$ is infinite, and will get a contradiction.

The Margulis Finiteness Theorem (see [Zi1], Theorem 8.1.2) states that a normal subgroup of Γ is either contained in the center $Z(G)$ of the semisimple Lie group G (in particular is finite) or is finite index in Γ. Since $\phi(\Gamma)$ is not finite then the kernel of ϕ must be finite. By the Selberg Lemma, Γ has a finite index subgroup Γ' which is torsion-free. Replacing Γ by Γ', we may assume that the kernel of ϕ is trivial, so that $\phi(\Gamma)$ is a subgroup of $\mathrm{Mod}(S)$. Henceforth we will denote this subgroup by Γ.

Let $\mathcal{PMF}(S)$ denote the space of projectived measured foliations on S (see [Th]). Recall that a subgroup $\Gamma < \mathrm{Mod}(S)$ is *nonelementary* if either of the following two equivalent (by [KM], Lemma 1.2.1) conditions holds:

- Γ fixes no finite subset of $\mathcal{PMF}(S)$.
- Γ contains a pair of *independent* pseudo-Anosov homeomorphisms h_1, h_2; that is, the sets of fixed points of h_1 and h_2 in $\mathcal{PMF}(S)$ are disjoint.

Theorem 2.4.1 of [KM] states that an irreducible lattice in a semisimple Lie group of rank at least two is not isomorphic to a nonelementary subgroup of $\mathrm{Mod}(S)$. Hence Γ is elementary. We note that, although the Theorem 2.4.1 of [KM] is stated only for closed surfaces, all of the arguments apply verbatim to prove the same result for punctured surfaces. But as noted above, if S has nonempty boundary then $\mathrm{Mod}(S)$ is isomorphic to the mapping class group of a punctured surface.

Note. Unlike in the case of discrete groups of isometries of hyperbolic space, there do exist examples of elementary groups of mapping classes which are nevertheless *nonamenable*. For example, the subgroup generated by Dehn twists about two intersecting curves that are disjoint from a third curve is elementary and nonamenable.

McCarthy and Papadapoulos (see Theorem 4.6 of [MP]), based on earlier work of Ivanov (see [Iv4, Iv3]), have classified subgroups of $\mathrm{Mod}(S)$ into four classes:

1. subgroups containing a pair of independent pseudo-Anosov elements (called *sufficiently large* subgroups in [MP]);
2. subgroups fixing the pair $\{F_-(g), F_+(g)\}$ of fixed points in $\mathcal{PMF}(S)$ for a certain pseudo-Anosov element $g \in \mathrm{Mod}(S)$ (such subgroups are virtually cyclic);
3. finite subgroups;
4. infinite subgroups leaving invariant a finite, nonempty, system of disjoint, nonperipheral, simple closed curves on S (such subgroups are called *reducible*).

Since Γ is elementary option 1 cannot occur. Since Γ is a lattice in a semisimple Lie group, options 2 and 3 clearly cannot occur (e.g., Γ is nonamenable). Hence Γ must be reducible, say leaving a finite collection C of disjoint simple closed curves invariant.

Since Γ leaves C invariant, each element of Γ gives a permutation of the connected components $\{S_1, \ldots, S_n\}, n > 0$ of $S \setminus C$. Hence there is a homomorphism $\Gamma \to H_n$ where H_n is the permutation group on n letters.

Let L be the kernel of this homomorphism. Then L, being finite index in Γ, is an irreducible, higher rank lattice. But L leaves each subsurface S_i invariant (here we are thinking of each S_i as a compact surface with boundary). Hence there is a homomorphism

$$\psi : L \to \mathrm{Mod}(S_1) \times \cdots \times \mathrm{Mod}(S_n)$$

with abelian kernel (see [BLM], Lemma 2.1), namely $\psi(g) = 1$ if and only if ψ is a product of Dehn twists about curves in C.

Applying the Margulis finiteness theorem to L, we see that the kernel of ψ must be finite, for otherwise it would be finite index and so L would be virtually abelian, which cannot happen since L is nonamenable. Again applying the Selberg Lemma we may, perhaps after passing to a finite index subgroup of L, assume that ψ has trivial kernel. In particular $\psi(L)$ is infinite.

Let $p_i : \mathrm{Mod}(S_1) \times \cdots \times \mathrm{Mod}(S_n) \to \mathrm{Mod}(S_i)$ be the natural projection. Then $p_j \circ \psi(L)$ is infinite for at least one $1 \leq j \leq n$. But by induction, since $\tau(S_j) < \tau(S)$, the homomorphism $p_j \circ \psi : L \to \mathrm{Mod}(S_j)$ must have finite image, a contradiction. $\diamond$

3 Proof of Theorem 1.4

We begin this section by proving that any finite group acts as a group of isometries of some surface. As noted above, this implies that some bound is needed in the statement of Theorem 1.4.

Proposition 3.1 (Finite groups act) *Let G be any finite group. Then there is some compact Riemann surface X so that G is a subgroup of the group of isometries of X.*

The following proof was suggested to us by R. Narasimhan.

Proof. Write $G = F/N$ where F is free on m generators and N is normal in F. Note that N is finitely generated since it is finite index in F. Let Y

be S^2 with $m+1$ points deleted. Then the unramified covering of Y with fundamental group N extends to a ramified covering X of S^2 on which G acts as the group of covering transformations. Of course the full group of isometries of X may be larger than G. $\diamond$

To prove Theorem 1.4 we first need an algebraic lemma.

Lemma 3.2 (Minimal $SL(n, \mathbf{Z}/k\mathbf{Z})$ quotients) *Let $n > 2, k > 1$ be integers. Then the smallest nontrivial quotient group of $SL(n, \mathbf{Z}/kZ)$ has order at least $2^{n(n-1)/2}\prod_{i=2}^{n-1}(2^i-1)$.*

Proof. Let $k = p_1^{r_1}\cdots p_s^{r_s}$ be the prime factorization of k. Then

$$SL(n, \mathbf{Z}/k\mathbf{Z}) = \prod_{i=1}^{s} SL(n, \mathbf{Z}/p_i^{r_i}\mathbf{Z})$$

Hence it clearly suffices to prove the theorem for $k = p^r$ for some prime p and integer $r > 0$. Let K be the kernel of the surjective homomorphism $\pi : SL(n, \mathbf{Z}/p^r\mathbf{Z}) \to SL(n, \mathbf{Z}/p\mathbf{Z})$. The group K can be thought of as the matrices in $G = SL(n, \mathbf{Z}/p^r\mathbf{Z})$ which are congruent to the identity matrix when the entries are taken mod p. By performing the binomial expansion on the expression $(I + pA)^{p^{r-1}}$, one may check that K is a p-group.

Now let K^+ denote the preimage under π of the center Z of $SL(n, \mathbf{Z}/p\mathbf{Z})$. Then Z is cyclic of order d, where d is the greatest common divisor of n and $p-1$. Now $G/K^+ \approx PSL(n, \mathbf{Z}/p\mathbf{Z})$, which is simple for $n \geq 3$ (see, e.g., [La], Theorem 9.3), so K^+ is a maximal normal subgroup of G.

We claim that K^+ is the unique maximal normal subgroup of $SL(n, \mathbf{Z}/p^r\mathbf{Z})$. If this were true then any nontrivial quotient of $SL(n, \mathbf{Z}/p^r\mathbf{Z})$ would have order at least

$$|PSL(n, \mathbf{Z}/p\mathbf{Z})| \geq |PSL(n, \mathbf{Z}/2\mathbf{Z})| = 2^{n(n-1)/2}\prod_{i=2}^{n-1}(2^i-1)$$

The final equality is an easy computation, and is in fact Exercise XIII.16 in [La].

To prove the claim, suppose H is a maximal normal subgroup of $G = SL(n, \mathbf{Z}/p^r\mathbf{Z})$, so that G/H is simple. Now $HK^+ \lhd G$, so if $H \neq K^+$ then by maximality $G = HK^+$. A basic isomorphism theorem gives $HK^+/H \approx K^+/K^+ \cap H$, so $K^+/K^+ \cap H \approx G/H$, a simple group.

Now K is solvable since it is a p-group, and $K^+/K = Z$ is cyclic of order d. Hence K^+ is a solvable group of order dp^m for some m. So G/H is simple and solvable, hence is abelian. But since $n \geq 3$, it is not hard to check that G is a perfect group, that is G equals its own commutator subgroup (by the same proof for fields, as in Theorem 9.2 of [La]), so that G has no nontrivial abelian quotients. Hence $G/H \approx K^+/K^+ \cap H$ is trivial and so $H = K^+$. $\diamond$

The following lemma seems to be well-known (see, e.g., [To], Corollary 1.4). We include a proof (implicit in [Ke]) in the smooth case.

Lemma 3.3 *Let $f \in Homeo(\Sigma_g)$ where $g \geq 2$. If f has finite order and is isotopic to the identity then f is the identity.*

The lemma isn't true for $g = 1$: there are periodic isometries of the flat torus which are isotopic to the identity but are not the identity.

Proof. Following [Ke], let h be any smooth Riemannian metric on Σ_g. Average the pullbacks of h by the powers of f. Then f acts conformally on this smooth, averaged metric. Now any conformal map on a smooth surface is complex-analytic, so by uniformization we have that the action of f is an isometric action for some hyperbolic metric on Σ_g. But any isometry on a hyperbolic surface isotopic to the identity is the identity, so we are done. $\diamond$

Now to prove the theorem. Let $\phi : SL(n, \mathbf{Z}) \to \text{Homeo}^+(\Sigma_g)$ be a homomorphism. Composing ϕ with the natural projection $\pi : \text{Homeo}(\Sigma_g) \to \text{Mod}(\Sigma_g)$, we claim that $\psi := \phi \circ \pi$ is trivial. By Theorem 1.1 ψ has finite image. The congruence subgroup property [BMS] implies that ψ factors through the homomorphism $SL(n, \mathbf{Z}) \to SL(n, \mathbf{Z}/k\mathbf{Z})$ for some integer $k > 1$. By Lemma 3.2, if the image of ψ were not trivial, then it would have order at least $2^{n(n-1)/2} \prod_{i=2}^{n-1}(2^i - 1)$.

But the Nielsen Realization Theorem (see [Ke]) shows that any finite subgroup of $\text{Mod}(S)$ can be reailzed as a group of isometries of some hyperbolic metric on S. Any such group has order at most $84(g-1)$ by Hurwitz's Theorem (see, e.g. [FK], Theorem V.1.3). This contradicts the assumption about g and n.

Hence ψ is trivial, so that every element of $\phi(SL(n, \mathbf{Z}))$ is isotopic to the identity. Choose any noncentral element $g \in SL(n, \mathbf{Z})$ of finite order. By Lemma 3.3 $\phi(g)$ must be the identity. Hence the normal subgroup N generated by g must act trivially on Σ_g. Since g is noncentral, by the Margulis finiteness theorem N must have finite index in $SL(n, \mathbf{Z})$, so that

ϕ must have finite image. But Lemma 3.3 then shows that every element must act trivially. $\diamond$

4 Braid groups

In this section we prove the analog of Theorem 1.1 for braid groups. We also extend the computation of the Poisson boundary of mapping class groups given in [KM] to braid groups.

4.1 Superrigidity

Let $B_n, n \geq 3$ denote the braid group on n strands.

Theorem 4.1 (Superrigidity with B_n target) *Let Γ be an irreducible lattice in a semisimple lie group G of $\mathbf{R}$-rank at least two. Then any homomorphism $\phi : \Gamma \to B_n$ has finite image.*

Proof. Let $\mathrm{Mod}(S_{n+1})$ be the mapping class group of the sphere S with $n+1$ punctures, and let $\mathrm{Mod}_x(S_{n+1})$ denote the subgroup which fixes a fixed puncture x. There is a short exact sequence

$$1 \to \mathbf{Z} \to B_n \to \mathrm{Mod}_x(S_{n+1}) \to 1$$

Let $\psi : \Gamma \to B_n$ be any homomorphism. Considering the exact sequence just given, by Theorem 1.1 and residual finiteness of Γ there is a finite index subgroup Γ' of Γ with $\psi(\Gamma') < \mathbf{Z}$. But since Γ' has Kazhdan's property T it follows (see [Zi1], 7.1.7) that $\psi(\Gamma')$, hence $\psi(\Gamma)$ is finite. $\diamond$

4.2 Poisson boundary of B_n

The paper [KM] was concerned with studying probablility measures and the associated random walks on the mapping class group. To any random walk there is an associated Poisson boundary. In [KM] conditions were given on the measure on the mapping class group to guarantee that the Poisson boundary is the Thurston boundary of Teichmuller space. These results were stated only for compact surfaces without boundary.

In fact, as mentioned above, the theorems and proofs hold verbatim if the surface has punctures. We are grateful to V. Kaimanovich for pointing out the following Theorem and proof, which is a consequence of this fact.

Fixing a finite set of generators for B_n provides a length function on B_n. Let $|g|$ denote the minimum length of a word in the generators representing g. There is a natural homomorphism $\pi : B_n \to \mathrm{Mod}(S_{n+1})$. Associated to S_{n+1} is the sphere $\mathcal{PMF}(S_{n+1})$ of projective measured foliations on S_{n+1} on which $\mathrm{Mod}(S_{n+1})$ and therefore B_n acts.

Theorem 4.2 (Poisson boundary of braid group) *Let μ be a probability measure on B_n such that the support of μ generates B_n. Suppose μ satisfies the finite first moment condition*

$$\sum_{g \in B_n} \mu(g)|g| < \infty$$

Then there is a unique μ-stationary measure ν on $\mathcal{PMF}(S_{n+1})$ such that $(\mathcal{PMF}(S_{n+1}), \nu)$ is the Poisson boundary of (B_n, μ). (see [KM] for definitions)

Proof. The map π projects μ to a measure $\pi_*\mu$ on $Mod(S_{n+1})$. The image of a set of generators of B_n is a set of generators for $Mod(S_{n+1})$. Since word length is nonincreasing with respect to the homomorphism π, it is clear that $\pi_*\mu$ has finite first moment with respect to the set of generators on $Mod(S_{n+1})$. Now by Corollary 2.3.2 of [KM] there is a unique $\pi_*\mu$-stationary measure ν on $\mathcal{PMF}(S_{n+1})$ such that $(\mathcal{PMF}(S_{n+1}), \nu)$ is the Poisson boundary of $(Mod(S_{n+1}), \pi_*\mu)$. However since B_n is a central extension of $Mod(S_{n+1})$ (the kernel $\mathbf{Z}$ of π is the center of B_n), by Theorems 2.1.5 and 5.2.7 of [Ka], the Poisson boundaries of (B_n, μ) and $(Mod(S_{n+1}), \pi_*\mu)$ coincide. $\diamond$

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