Day 4 Lecture 3.

Dictionary

H_M	M	Comment
$V_1 \oplus V_2$	$M_1 \# M_2$	Freedman
$O(H_M) =$	$\mathrm{Mod}(M)$	Freedman – Quinn
$v \in H_M$, $v^2 = 0$	v = fiber of elliptic fibration ($\exists!$ up to top isotopy)	Farb-Looijenga
Ref_v, $v^2 = -2$	T_S	F–L
$\varphi \in \mathrm{Stab}(v)$, $v^2 = 0$	$f \in \mathrm{Diff}^+(M)$ elliptic fiber-preserving	F–L

Thm (F-L) Let $M = Bl_9(\mathbb{P}^2)$ (or $M = K3$)

Let $\varphi \in \mathrm{Mod}(M)$ s.t. φ is a parabolic type and in $v^\perp / \mathbb{Z}v$

($\varphi \in \mathrm{Stab}(v) : v^2 = 0$)

$\Rightarrow \exists$ smooth elliptic fibering $M \xrightarrow{\pi} \mathbb{P}^1$ (unique up to top isotopy)

and f that's fiber preserving & translation on fiber s.t. $f_* = \varphi$.

This gives a realisation of
$$\begin{tikzcd} \mathbb{Z}^8 \arrow[r] \arrow[dr, dashed] & \mathrm{Mod}(M) \\ & \mathrm{Diff}(M) \arrow[u] \end{tikzcd}$$

$B_{12}(S^2) \dashrightarrow \mathrm{Mod}(M)$

base diffeomorphism of $M \to \mathbb{CP}^1 \rightsquigarrow$ lift to M.

Define more sections of $Bl_9(\mathbb{CP}^2) \to \mathbb{CP}^1$

considering conic over 5 points, and line over 2 sections.

K3 story:

$$0 \to v^{\perp}/v \hookrightarrow \mathrm{Stab}([v]) \twoheadrightarrow O(v^{\perp}/v) \to 1$$

$$\mathbb{Z}^{20} \qquad\qquad O(2,18)(\mathbb{Z})$$

Kodaira: all K3's are diffeomorphic. (Def of K3, nowhere zero 2-form on complex surface $\pi_1 = 0$)

Kummer, quartic in $\mathbb{CP}^3$, elliptic fiber, branched cover of $\mathbb{CP}^2$.

Given $\varphi \in \mathrm{Mod}(M)_{K3} = O(H_M)$

- φ finite order (F-L) Nielsen Realisation of K3.
- φ parabolic
- φ hyperbolic (semisimple) ∞ order (Example: McMullen Dynamics on K3)

Q: Which finite order in Mod(M) are realizable as Diff or Homeo or Isom.

A: • T_s is not realizable by diffeos.

• (F-L) all different in this case.

<u>Pf idea:</u> $\mathrm{Mod}(M) \circlearrowright \mathrm{Teich}(M)$ curvature ≤ 0.

Fixed point theory ....