

Dictionary

Hm

M

Comment

$V_1 \oplus V_2$

$M_1 \# M_2$

Freedman

$O(Hm) = Mod(M)$

Freedman - Quinn

$v \in Hm$   
 $v^2 = 0$

$v =$  fiber of elliptic fibration  
( $\exists!$  up to top isotopy)

Farb - Looijenga

Refs  
 $v^2 = -2$

$T_S$

F-L

$\varphi \in Stab(v)$   
 $v^2 = 0$

$f \in Diff^+(M)$   
elliptic fiber-preserving

F-L

Thm(F-L) Let  $M = Bl_9(\mathbb{P}^2)$  (or  $M = K3$ )

Let  $\varphi \in Mod(M)$  s.t  $\varphi$  is a parabolic type and in  $V^1/2V$   
( $\varphi \in Stab(v) : v^2 = 0$ )

$\Rightarrow \exists$  smooth elliptic fibering  $M \xrightarrow{\pi} \mathbb{P}^1$  (unique up to top isotopy)  
and  $f$  that's fiber preserving s.t  $f|_{\text{fiber}} = \varphi$ .  
& translation on fiber.

This gives a realisation of  $\mathbb{Z}^8 \rightarrow \text{Mod}(M)$   
 $\swarrow \quad \uparrow$   
 $\text{Diff}(M)$

$$B_{12}(S^2) \dashrightarrow \text{Mod}(M)$$

base diffeomorphism of  $M \rightarrow \mathbb{CP}^1 \rightsquigarrow$  lift to  $M$ .

Define more sections of  $Bl_9(\mathbb{CP}^2) \rightarrow \mathbb{CP}^1$

considering conic over 5 points, and line over 2 sections.

K3 story:

$$0 \rightarrow \mathbb{Z}^{20} \rightarrow \text{Stab}(\mathbb{Z}^3) \rightarrow O(\mathbb{Z}^2/\mathbb{Z}) \rightarrow 1$$

$$O(2, 18)(\mathbb{Z})$$

Kodaira: all K3's are diffeomorphic. (Def of K3, nontrivial zero 2-form on complex surface  $\pi_1 = 0$ )  
 $\swarrow$   
 Kummer, quartic in  $\mathbb{CP}^3$ , elliptic fiber, branched over  $\mathbb{CP}^2$ .

Given  $\varphi \in \text{Mod}(M) \stackrel{K3}{=} O(M)$

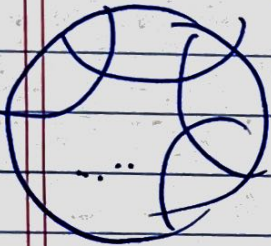
- $\varphi$  finite order (F-L) Nielsen Realization of K3.
- $\varphi$  parabolic
- $\varphi$  hyperbolic (semisimple)  $\infty$  order (Example: McMullen Dynamics on K3)

Q: Which finite order in  $\text{Mod}(M)$   
are realizable as Diff or Homeo or Isom.

A: •  $T_5$  is not realizable by diffeos.

- (F-L) all different in this case.

Pf idea:  $\text{Mod}(M) \rightarrow \text{Teich}(M)$  curvature  $\leq 0$ .



Fixed point theory . . . . .