CLASSICAL TESSELLATIONS AND 3-MANIFOLDS, SPRING 2014, HOMEWORK 4

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Homework is assigned on Fridays; it is due at the start of class the week after it is assigned. So this homework is due May 2nd.

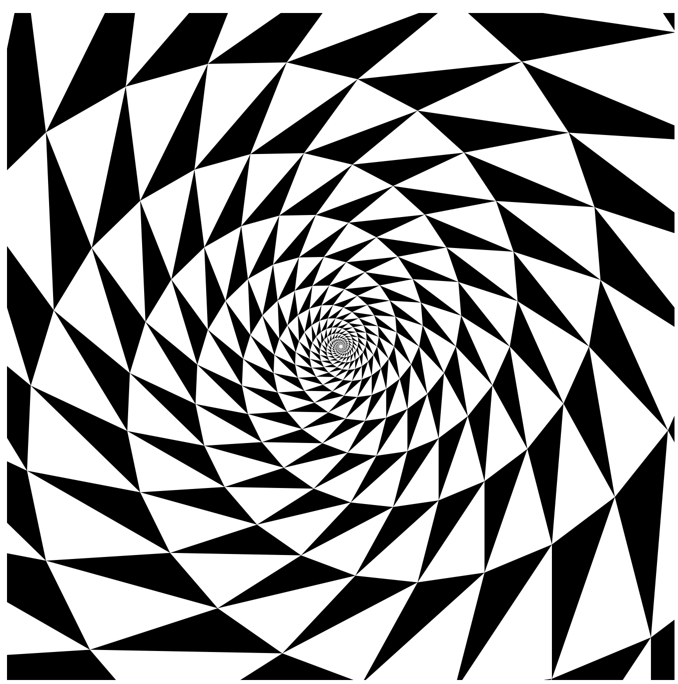
Problem 1. For each of the 17 Euclidean wallpaper groups (i.e. the groups of discrete cocompact isometries of the Euclidean plane)

(1) draw the quotient orbifold;
(2) exhibit the orbifold as the quotient of a torus by a finite group; and
(3) draw a periodic tessellation of the plane with exactly this group of symmetries.

Problem 2. Suppose that T is a Euclidean triangle. Show that you can make a polyhedral 3-dimensional tetrahedron (*not* assumed to be regular!) with 4 faces all exactly in the shape of T, if and only if the angles of T are all acute. What is the group of symmetries of your tetrahedron? What is the associated spherical orbifold?

Build one out of paper for T a triangle with side lengths in the proportion $4:5:6$ and take a photo.

Problem 3. The Figure shows (part of) a symmetric tiling of the plane minus one point. What is the group of symmetries of the tiling? (here some of the symmetries might not be isometries, but rather *similarities* of the plane — transformations that change size but preserve shape). Is the group discrete, and does it act cocompactly on the plane minus a point? Draw a picture of the quotient. What is the quotient?

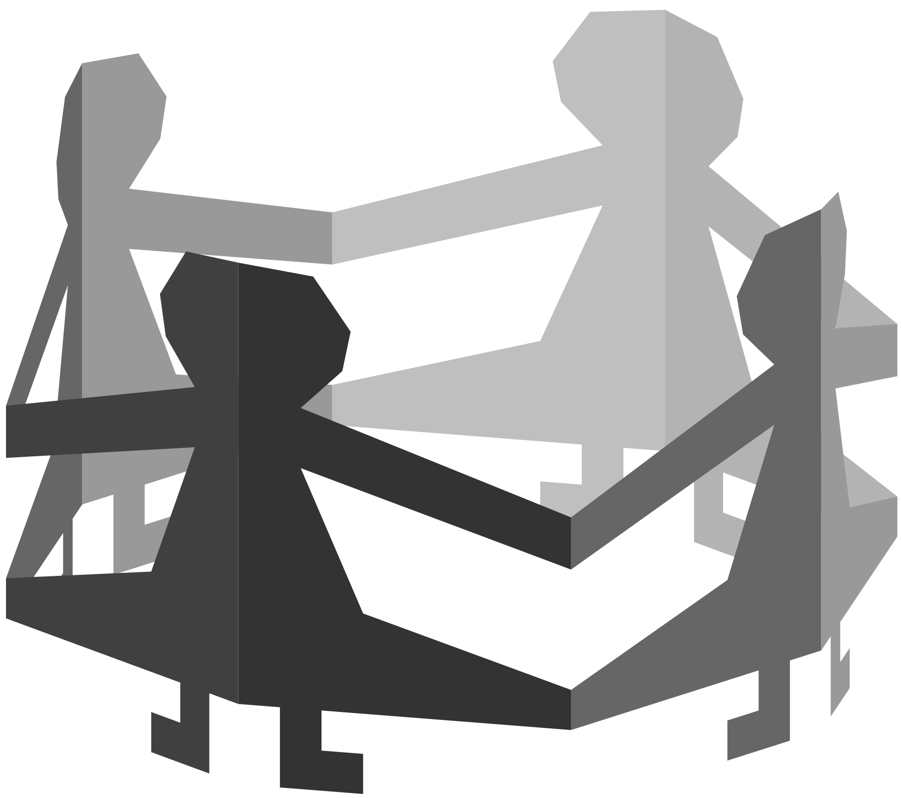


Problem 4. Show that you can't tile 3-dimensional space with regular tetrahedra. On the other hand, show that you *can* tile 3-dimensional space with regular tetrahedra *and* regular octahedra. Do you need more tetrahedra or more octahedra? Why?

Problem 5. Show that you can inscribe a regular cube in a regular dodecahedron in such a way that the vertices of the cube are (a subset of the) vertices of the dodecahedron. If the cube has vertices at $(\pm 1, \pm 1, \pm 1)$, what are the coordinates of the vertices of the dodecahedron?

Problem 6. Show that any finite group acts by isometries on a sphere of some dimension.

Problem 7 (Thurston). Take a flat rectangle of paper, and glue two opposite ends to make a cylinder. Fold up the cylinder somehow so that it lies flat in the plane, and cut pieces out of the folded cylinder with scissors in such a way that when you unfold it back to a cylinder, what is left looks like this:



(take a photo!)

Interpret this in terms of orbifolds and a finite group of symmetries acting on the cylinder.

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