CLASSICAL TESSELLATIONS AND 3-MANIFOLDS, SPRING 2014, HOMEWORK 1

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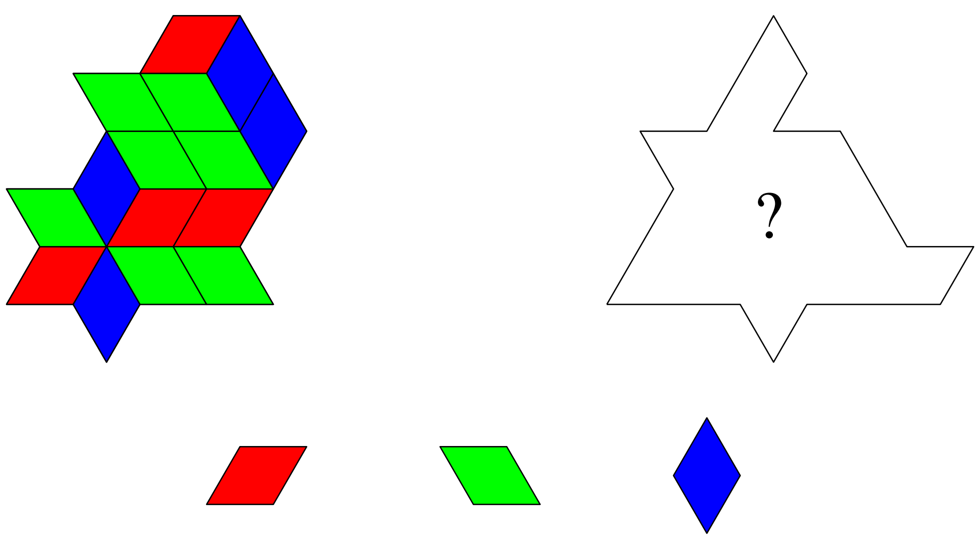
Homework is assigned on Fridays; it is due at the start of class the week after it is assigned. So this homework is due April 11th.

Problem 1. How can you draw a good picture of a soccerball? Explain what qualities the picture should have, and draw one.

Problem 2. Draw a picture of a Cayley graph of the alternating group A_5 with generators $a = (12)(34)$ and $b = (12345)$. Note that they satisfy relations $a^2 = 1$, $b^5 = 1$ and $(ab)^3 = 1$. So you could build it out of $ababab$ hexagons and $bbbbb$ pentagons.

Problem 3. For which positive integers $n > 2$ can you tile the plane with regular n-gons?

Problem 4. The region on the left is tiled by red, green and blue lozenges of a fixed size and orientation, without overlaps. Can the region on the right be tiled by lozenges? Give three different examples of regions that can't be tiled by lozenges, for three different reasons.



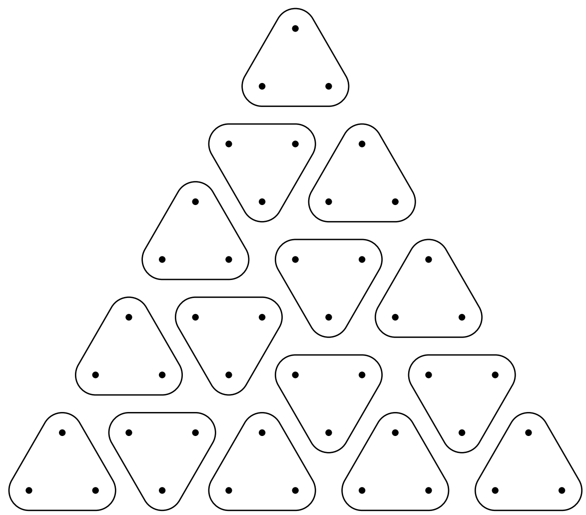
Problem 5. Show that every isometry (i.e. distance-preserving symmetry) of the Euclidean plane is a product of at most 3 reflections. Is 3 the minimum necessary?

Problem 6. Find an interesting example of a tiled region, and take a photo of it. Then describe the tiling in terms that would make sense to a blind person.

Problem 7. Let S be a unit square in the plane with one edge parallel to the x axis, and let S' be a unit square in the plane with one edge making a $60°$ angle with the x axis. Show that you can cut S up into finitely many pieces which can be translated to make the square S'. How many pieces do you need?

Problem 8 (Napoleon's Theorem). Let T be a triangle in the plane. Let E_1, E_2, E_3 be three equilateral triangles constructed on the edges of T (and on the outside). Show that the centers of the E_i form an equilateral triangle. Use this fact to produce a periodic tessellation of the plane from (rotated and translated) copies of T and copies of the E_i. Draw a good picture of such a tessellation.

Problem 9. Consider a triangular array of dots, with N dots on each side. When is it possible to subdivide this array into disjoint triangular arrays of dots with 2 on each side? The figure shows a positive solution when $N = 9$. Answer this question for all $2 \leq N \leq 12$.



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