Math151a, Fall 2008

1. Let F be the free group on two elements a, b. In the sequel, for clarity, we denote a^{-1} by A and b^{-1} by B. Let u_i for $i = 1 \cdots 6$ be the following elements: $u_1 = abaB$, $u_2 = ABAbaBabAbaBABAbaBababABB$, $u_3 = ABAba$, $u_4 = BabAbaBABAbba$, $u_5 = BabABababA$, $u_6 = aaBAAb$. Let G be the subgroup of F generated by the u_i.

   (a) Show that G is free of rank 6

   (b) Given elements x, y in a group, the *commutator*, denoted $[x, y]$, is the element $[x, y] = xyx^{-1}y^{-1}$. Show that $[u_1, u_2][u_3, u_4][u_5, u_6] = (abaBABAbaBabAA)^3$

   (c) Let $v = abaBABAbaBabAA$, and let H be the subgroup of F generated by the u_i together with v. What is the rank of H?