

CLASSICAL GEOMETRY — SYLLABUS

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1. A CRASH COURSE IN GROUP THEORY

1.1. Basic examples and definitions.

1.1.1. *Cyclic groups, Dihedral groups, symmetric groups.*

1.2. Products of groups, subgroups, normal subgroups.

1.3. Homomorphisms, exact sequences.

1.4. Commutators, centers, Abelian groups, Nilpotent groups, Solvable groups.

1.5. Automorphisms (inner and outer), semi-direct products.

1.6. Free groups, generators and relations, finitely generated/presented groups.

1.7. Topological groups, Lie groups.

2. MODEL GEOMETRIES IN DIMENSION TWO

2.1. The Euclidean plane.

2.1.1. *Euclid's axioms.*

2.1.2. *A closer look at the fourth postulate.*

2.1.3. *A closer look at the parallel postulate.*

2.1.4. *Symmetries of \mathbb{E}^2 .*

2.2. The 2-sphere.

2.2.1. *Elliptic geometry.*

2.2.2. *Spherical trigonometry.*

2.2.3. *The area of a spherical triangle.*

2.2.4. *Kissing numbers — the Newton–Gregory problem.*

2.2.5. *Reflections, rotations, involutions; $SO(3)$.*

2.2.6. *Algebraic groups.*

2.2.7. *Quaternions and the group \mathbb{S}^3 .*

2.3. The hyperbolic plane.

2.3.1. *The problem of models.*

2.3.2. *The Poincaré (conformal) model.*

- 2.3.3. *The upper half-space model.*
- 2.3.4. *The Klein (projective) model.*
- 2.3.5. *Hyperbolic trigonometry.*
- 2.3.6. *The area of a hyperbolic triangle.*
- 2.3.7. *Projective geometry.*
- 2.3.8. *Elliptic, parabolic, hyperbolic isometries.*
- 2.3.9. *Horocircular geometry.*

3. TESSELLATIONS IN DIMENSION TWO

3.1. **The topology of surfaces.**

- 3.1.1. *Gluing polygons.*
- 3.1.2. *The fundamental group and covering spaces.*
- 3.1.3. *Discrete groups.*

3.2. **Discrete subgroups of $\text{Isom}^+(\mathbb{E}^2)$.**

- 3.2.1. *Integral quadratic forms.*
- 3.2.2. *Continued fractions and $SL(2, \mathbb{Z})$.*
- 3.2.3. *Aperiodic tilings.*
- 3.2.4. *Penrose tiles.*
- 3.2.5. *Foliations and laminations.*
- 3.2.6. *Euclidean wallpaper groups.*
- 3.2.7. *Moduli of Euclidean orbifolds.*

3.3. **Finite subgroups of $SO(3)$ and \mathbb{S}^3 .**

- 3.3.1. *The “fair dice”.*
- 3.3.2. *Spherical orbifolds.*
- 3.3.3. *Reflection groups, Coxeter diagrams.*
- 3.3.4. *“Bad” orbifolds.*

3.4. **Discrete subgroups of $PSL(2, \mathbb{R})$.**

- 3.4.1. *Mapping-class groups, Teichmüller space.*
- 3.4.2. *Dehn twists and Lickorish’s theorem.*

4. APPENDIX — WHAT IS GEOMETRY?

4.1. **Klein’s “Erlanger Programm”.**

- 4.1.1. *Category theory.*

4.2. **Metric geometry.**

4.2.1. *Metric spaces.*

4.2.2. *Isometries, Lipschitz maps.*

4.3. **Differential geometry.**

4.3.1. *Smooth Manifolds.*

4.4. **Topology.**

4.4.1. *Open and closed sets, continuous maps.*

4.4.2. *Subspace topology, quotient topology.*

4.4.3. *Homeomorphisms.*

4.4.4. *Connected, locally connected, compact, locally compact.*

4.4.5. *Hausdorff.*

4.4.6. *Manifolds.*

REFERENCES

- [1] J. Montesinos, *Classical tessellations and three-manifolds*, Springer-Verlag Universitext (1987)
- [2] J. Stallings, *Category language*, notes; available at <http://www.math.berkeley.edu/~stall>
- [3] W. Thurston, *Three-dimensional geometry and topology, vol. 1*, Princeton University Press, Princeton Math. Series 35 (1997)

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