

Math 13200 – Section 58 – Midterm 1

April 13, 2016

Information and Directions

- This exam will last 50 minutes.
- This is a closed-book exam.
- No electronic devices are allowed to be used during this exam.
- **Partial credit is given for showing your calculations and explaining your thoughts.**

Name: SOLUTIONS

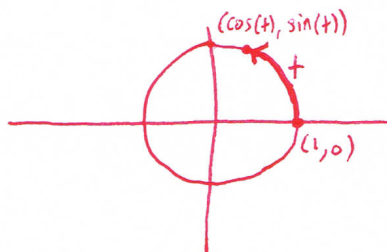
Problem	Points	Score
1	8	
2	4	
3	16	
4	16	
5	16	
6	20	
7	20	

Problem 1 (8 points)

Define $\cos(t)$ and $\sin(t)$ based on moving around the unit circle.

Be detailed.

If we start at the point $(1, 0)$ on the unit circle and travel counterclockwise a distance of t , then the x -coordinate of the point we end up at is $\cos(t)$, and the y -coordinate of the point we end up at is $\sin(t)$.



Problem 2 (4 points)

What is $\csc(\pi/4)$?

Since $\csc(x) = \frac{1}{\sin(x)}$, we get

$$\csc(\pi/4) = \frac{1}{\sin(\pi/4)} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$$

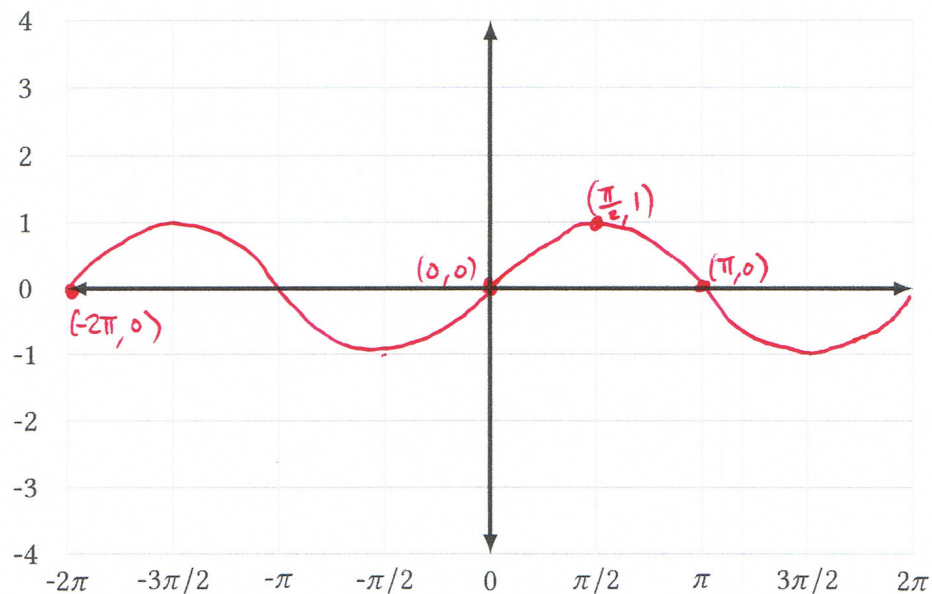
Problem 3 (16 points)

Part 1 (8 points)

Draw the graph of the equation

$$y = \sin(x)$$

For at least three different points on your graph, label the point with its x - and y -coordinates.

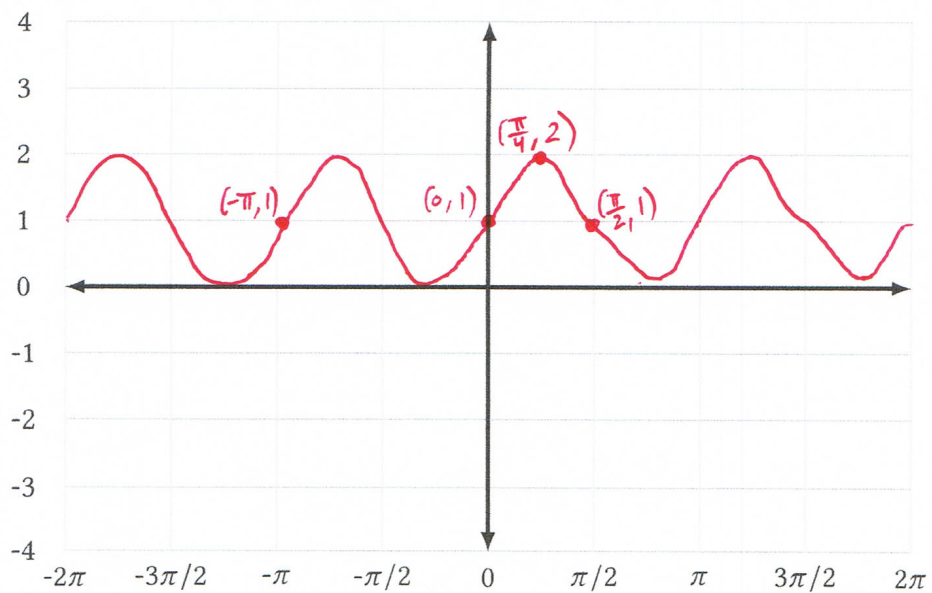


Part 2 (8 points)

Draw the graph of the equation

$$y = \sin(2x) + 1$$

For at least three different points on your graph, label the point with its x - and y -coordinates.



Problem 4 (16 points)

What is the equation of the tangent line to $y = \cos(x)$ at $x = \pi/3$?

slope of tangent line = derivative of $\cos(x)$ at $\frac{\pi}{3}$

$$\frac{d}{dx}(\cos(x)) = -\sin(x), \quad \text{and} \quad -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

point the tangent line goes through = $\left(\frac{\pi}{3}, \cos\left(\frac{\pi}{3}\right)\right) = \left(\frac{\pi}{3}, \frac{1}{2}\right)$

point-slope formula: $y - \frac{1}{2} = \left(-\frac{\sqrt{3}}{2}\right)\left(x - \frac{\pi}{3}\right)$

$$y = \left(-\frac{\sqrt{3}}{2}\right)x + \left(\frac{\pi}{2\sqrt{3}} + \frac{1}{2}\right)$$

Problem 5 (16 points)

What is the derivative of $\sin(x^3)$?

By the chain rule,

$$\frac{d}{dx}(\sin(x^3)) = \cos(x^3) \cdot \frac{d}{dx}(x^3) = 3x^2 \cos(x^3)$$

Problem 6 (20 points)

For this problem, define the function

$$f(x) = x^3 - 12x$$

Part 1 (10 points)

Find the critical points of f on the interval $[-3, 5]$.

For each of them, specify which kind of critical point it is (remember, there are three kinds).

c is a critical point of f on an interval if :

- it is an end point of the interval
- it is a stationary point (i.e., $f'(c) = 0$)
- it is a singular point (i.e., $f'(c)$ does not exist)

The end points of $[-3, 5]$ are -3 and 5 .

The derivative of f is

$$f'(x) = 3x^2 - 12$$

which always exists, so there are no singular points, but

$$f'(x) = 3x^2 - 12 = 0$$

$$x^2 - 4 = 0$$

$$(x-2)(x+2) = 0$$

so there are two stationary points, 2 and -2 (both of which are in the interval we're looking at, $[-3, 5]$.)

Thus the critical points of f on $[-3, 5]$ are

-3 (end point), -2 (stationary point), 2 (stationary point), 5 (end point)

Part 2 (10 points)

(Remember, the function f was defined at the beginning of this problem.)

Find the maximum and minimum value that f reaches on the interval $[-3, 5]$.

By the Critical Value Theorem, any maximum or minimum value that occurs must occur at a critical point. Because f is continuous, the Max-Min Existence Theorem says that a maximum and a minimum must occur.

Therefore, we only need to look at each critical point and find which has the largest and smallest value of f .

These will be the maximum and minimum of f on the interval.

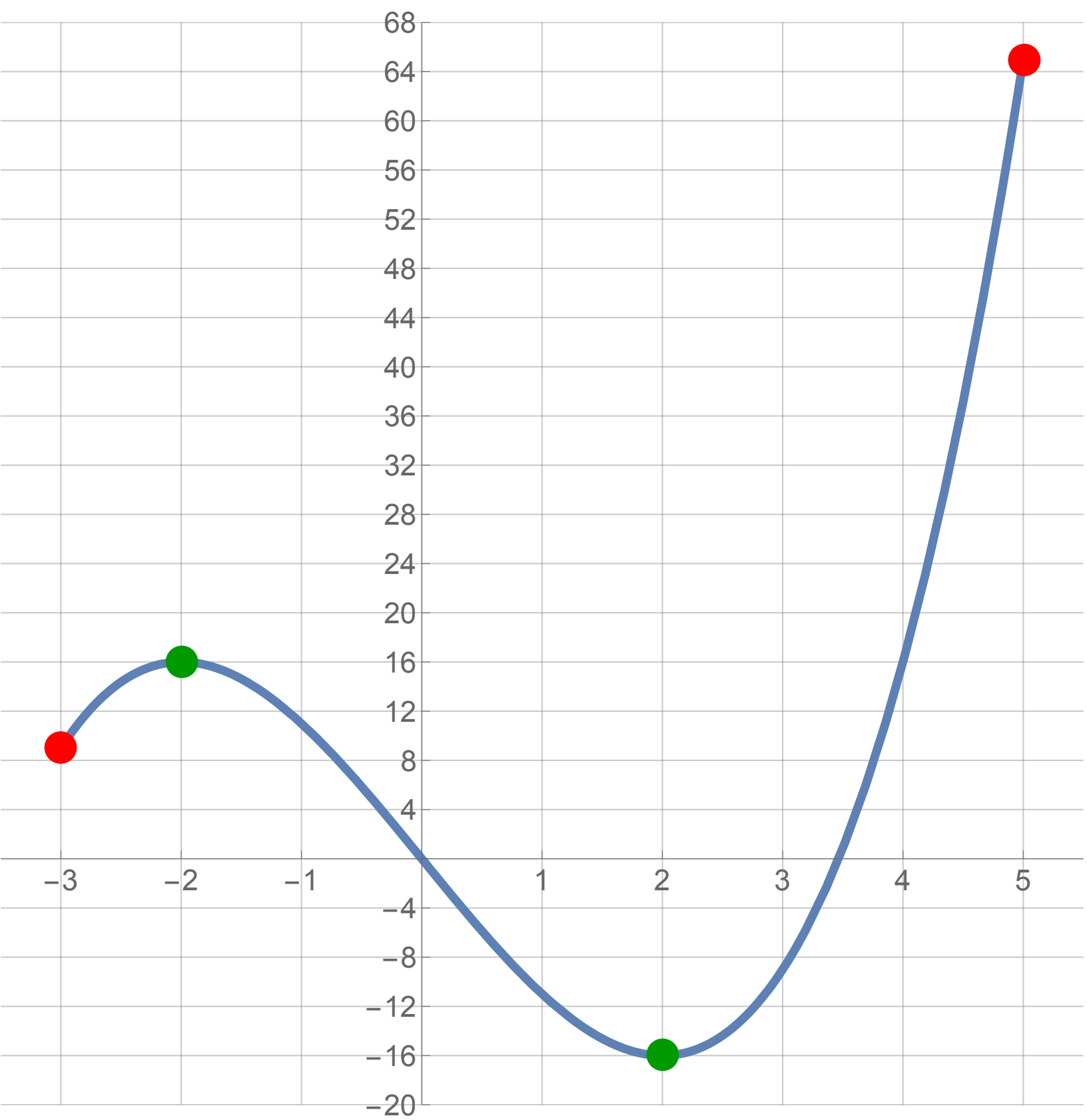
$$f(-3) = (-3)^3 - 12(-3) = -27 + 36 = 9$$

$$f(-2) = (-2)^3 - 12(-2) = -8 + 24 = 16$$

$$f(2) = 2^3 - 12(2) = 8 - 24 = -16 \quad \text{smallest}$$

$$f(5) = 5^3 - 12(5) = 125 - 60 = 65 \quad \text{largest}$$

Therefore the minimum value of f on $[-3, 5]$ is -16 , and the maximum value of f on $[-3, 5]$ is 65 .



Problem 7 (20 points)

For this problem, define the function

$$g(x) = -2x^3 + 6x^2 + 18x$$

Part 1 (10 points)

Where is g decreasing?

(Hint: Find g' and use the "Monotonicity Theorem". Your answer should be an interval or intervals.)

$$g'(x) = -6x^2 + 12x + 18$$

By the Monotonicity Theorem, g is decreasing when $g'(x) < 0$.

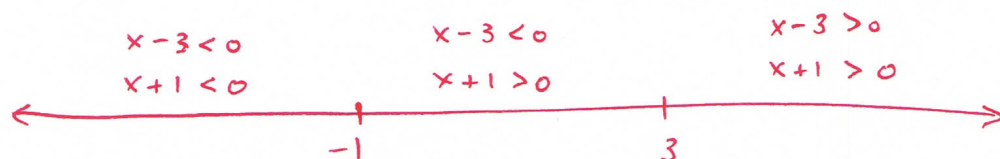
Thus, we solve

$$-6x^2 + 12x + 18 < 0$$

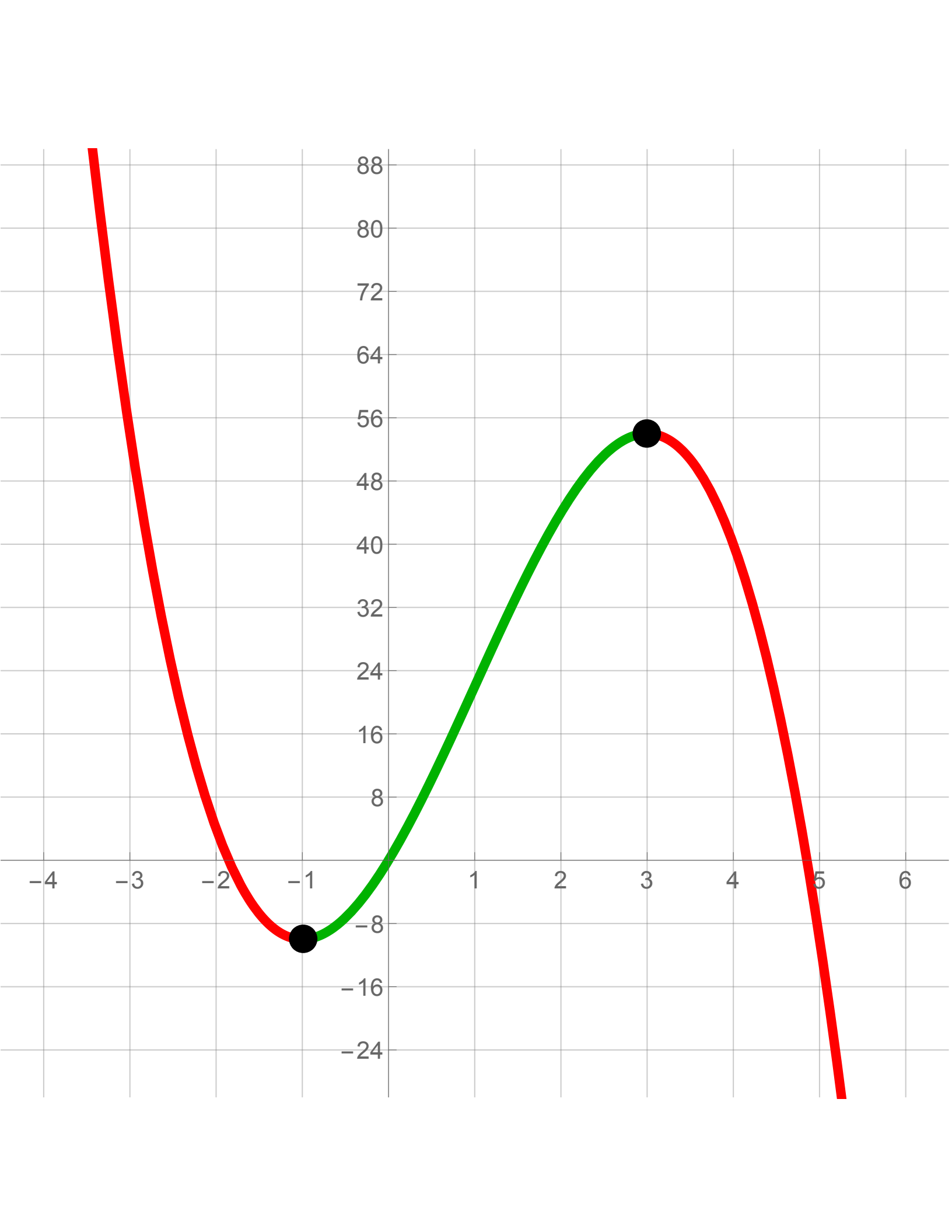
$$-x^2 + 2x + 3 < 0$$

$$x^2 - 2x - 3 > 0$$

$$(x-3)(x+1) > 0$$



Thus, $g'(x) < 0$ when $(x-3)(x+1) > 0$, which occurs when $x < -1$ and when $x > 3$. In other words, g is decreasing on the intervals $(-\infty, -1)$ and $(3, \infty)$.



Part 2 (10 points)

(Remember, the function g was defined at the beginning of this problem.)

Where is g concave up?

(Hint: Find g'' and use the "Concavity Theorem". Your answer should be an interval or intervals.)

$$g''(x) = -12x + 12$$

By the Concavity Theorem, g is concave up when $g''(x) > 0$.

Thus, we solve

$$-12x + 12 > 0$$

$$-12x > -12$$

$$x < 1$$

Thus, $g''(x) > 0$ when $x < 1$. In other words,

g is concave up on $(-\infty, 1)$.

