

Math 13200 – Section 58 – Final Exam

June 9, 2016

Information and Directions

- This exam will last 120 minutes.
- This is a closed-book exam.
- No electronic devices are allowed to be used during this exam.
- **Partial credit is given for showing your calculations and explaining your thoughts.**

Name: _____

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	

Problem 1 (10 points)**Part 1 (5 points)**

What is the definition of $\ln(x)$? For which real numbers x is it defined?

Part 2 (5 points)

If $\ln(a) = 3$ and $\ln(b) = 5$, then what is $\ln(ab)$?

Problem 2 (10 points)

Part 1 (5 points)

What is the equation of the tangent line to $y = \sin(x)$ at $x = \frac{\pi}{6}$?

Part 2 (5 points)

What is the equation of the tangent line to $y = \sin(\ln(x))$ at $x = 1$?

Problem 3 (10 points)

Part 1 (5 points)

Find the derivative $\frac{d}{dx} x \cos(x^2)$.

Part 2 (5 points)

Find the antiderivative $\int x \cos(x^2) dx$.

Problem 4 (10 points)

Part 1 (5 points)

Find the derivative $\frac{d}{dx} e^{\sin(x)} \cos(x)$.

Part 2 (5 points)

Find the antiderivative $\int e^{\sin(x)} \cos(x) dx$.

Problem 5 (10 points)

For this problem, define the function

$$G(x) = \int_{17}^x e^{\cos(t)} dt$$

Part 1 (5 points)

What is $G'(x)$?

Part 2 (5 points)

Is G concave up, concave down, or neither at the point $x = \frac{\pi}{2}$?

Problem 6 (10 points)

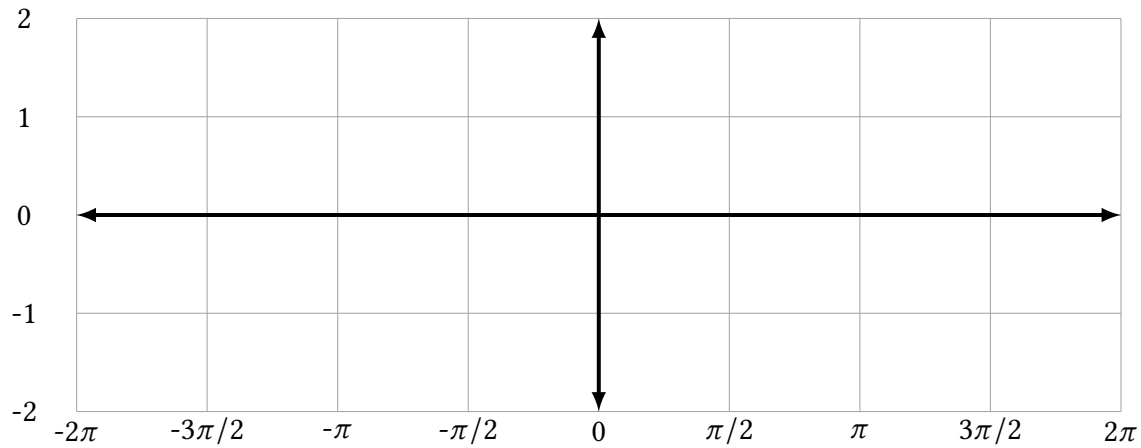
Find the definite integral $\int_{\ln(3)}^{\ln(8)} e^x \sqrt{e^x + 1} \, dx$. For this problem, you must simplify fully.

Problem 7 (10 points)

Consider the region bounded by the curves $y = 2 \sin(x)$ and $y = 2 \cos(x)$, between their intersection at $x = \frac{\pi}{4}$ and their intersection at $x = \frac{5\pi}{4}$.

Part 1 (5 points)

Plot the region on this grid. For each of the two intersections described above, label the point with its x - and y -coordinates.



Part 2 (5 points)

Find the area of the region.

Problem 8 (10 points)

Consider the region bounded by the lines $y = 0$, $x = 0$, and $x = 1$, and the curve $y = e^{x^2}$.

If we rotate the region around the y -axis, what is the volume of the resulting shape?

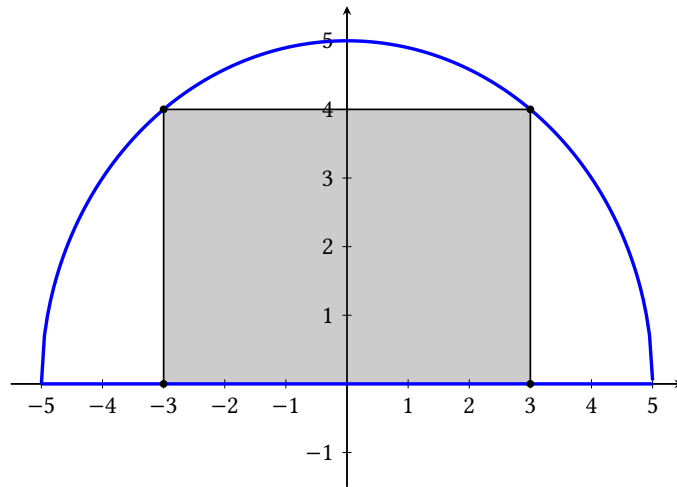
Problem 9 (10 points)

Consider the region bounded by the line $y = \frac{1}{2}x$ and the curve $y = \sqrt{x}$.

If a shape has this region as its base, and every cross-section of the shape which is perpendicular to the x -axis is a square, what is the volume of the shape?

Problem 10 (10 points)

Suppose that I want to make a rectangle with the largest area I can, but it is required that the rectangle sits inside the semicircle $y = \sqrt{25 - x^2}$ (see the picture below).



What are the width and the height of the rectangle I should choose to get the maximum area?