

Math 13200 – Section 58 – Final Exam Sample

June 4, 2016

Information and Directions

- This exam will last 120 minutes.
- This is a closed-book exam.
- No electronic devices are allowed to be used during this exam.
- **Partial credit is given for showing your calculations and explaining your thoughts.**

Name: _____

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	

Problem 1 (10 points)

Part 1 (5 points)

What is the definition of the number e ?

Part 2 (5 points)

What is $e^{\ln(2)+\ln(7)}$?

Problem 2 (10 points)

Part 1 (5 points)

What is the equation of the tangent line to $y = \cos(x)$ at $x = \frac{\pi}{4}$?

Part 2 (5 points)

What is the equation of the tangent line to $y = \ln(\sin(x))$ at $x = \frac{\pi}{2}$?

Problem 3 (10 points)

Part 1 (5 points)

Find the derivative $\frac{d}{dx} \sin^3(x) \cos(x)$.

Part 2 (5 points)

Find the antiderivative $\int \sin^3(x) \cos(x) dx$.

Problem 4 (10 points)

Part 1 (5 points)

Find the derivative $\frac{d}{dx} e^x \sin(e^x)$.

Part 2 (5 points)

Find the antiderivative $\int e^x \sin(e^x) dx$.

Problem 5 (10 points)

For this problem, define the function

$$G(x) = \int_{13}^x \ln(t^2 + 1) \, dt$$

Part 1 (5 points)

What is $G'(x)$?

Part 2 (5 points)

Is G concave up, concave down, or neither at the point $x = 2$?

Problem 6 (10 points)

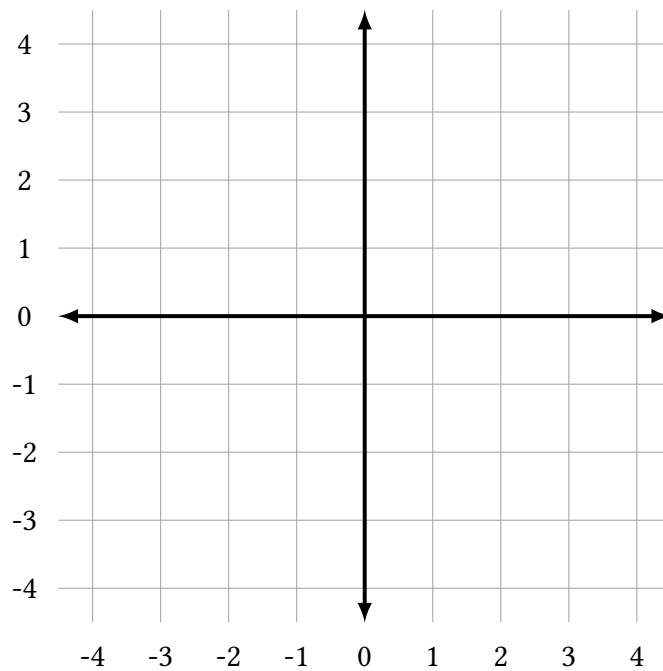
Find the definite integral $\int_1^{e^3} \frac{\ln(x)}{x} dx$. For this problem, you must simplify fully.

Problem 7 (10 points)

Consider the region bounded by the lines $x = 0$ and $y = ex$, and the curve $y = e^x$.

Part 1 (5 points)

Plot the region on this grid. For each intersection between the boundaries described above, label the point with its x - and y -coordinates. (Remember, $e \approx 2.7$.)



Part 2 (5 points)

Find the area of the region.

Problem 8 (10 points)

Consider the region bounded by the lines $y = 0$, $x = 0$, and $x = \sqrt{\pi}$, and the curve $y = \sin(x^2)$. If we rotate the region around the y -axis, what is the volume of the resulting shape?

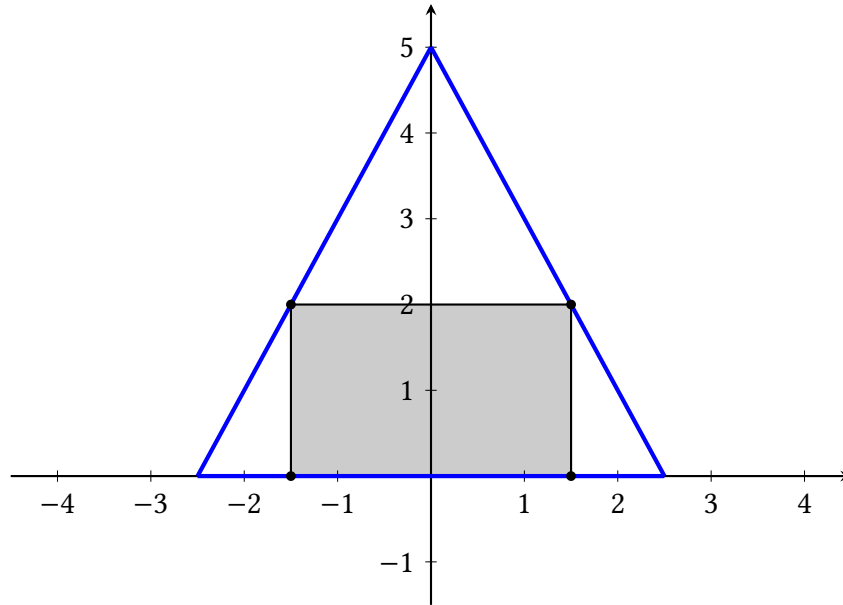
Problem 9 (10 points)

Consider the region bounded by the line $y = 2x$ and the curve $y = x^2$.

If a shape has this region as its base, and every cross-section of the shape which is perpendicular to the x -axis is a circle, what is the volume of the shape?

Problem 10 (10 points)

Suppose that I want to make a rectangle with the largest area I can, but it is required that the rectangle sits inside the triangle formed by $y = 0$, $y = -2x + 5$, and $y = 2x + 5$ (see the picture below).



What are the width and the height of the rectangle I should choose to get the maximum area?