

What does " $\lim_{x \rightarrow c} f(x) = L$ " mean?

For any there exists some
measurement of how close we want $f(x)$ to be to L measurement of how close to require x to be to c

Such that $\Rightarrow$
whenever x is within the chosen distance away from c (though not equal to c itself) the corresponding $f(x)$ is within the desired distance away from L

Examples

" $\lim_{x \rightarrow \infty} f(x) = 2$ " means

For any $\epsilon > 0$, there exists some $M > 0$

Such that $x > M \Rightarrow |f(x) - 2| < \epsilon$

" $\lim_{x \rightarrow 3} f(x) = -\infty$ " means

For any $N > 0$, there exists some $\delta > 0$

such that $0 < |x - 3| < \delta \Rightarrow f(x) < -N$

" $\lim_{x \rightarrow 5} f(x) = 13$ " means

For any $\epsilon > 0$, there exists some $\delta > 0$

such that $0 < |x - 5| < \delta \Rightarrow |f(x) - 13| < \epsilon$