

Math 13100 – Section 58 – Midterm 3

March 4, 2016

Information and Directions

- This exam will last 50 minutes.
- This is a closed-book exam.
- No electronic devices are allowed to be used during this exam.
- Partial credit is given for showing your calculations and explaining your thoughts.

Name: _____

Problem	Points	Score
1	30	
2	40	
3	30	

Problem 1 (30 points)

Part 1 (10 points)

Let x be a real number, and let f be a function. Give the precise definition (using a limit) of the derivative of f at x :

$$f'(x) =$$

lim

Part 2 (10 points)

Using only the definition you wrote above, and your experience with computing limits, compute the derivative $f'(x)$, where $f(x) = x^2$. Don't use the derivative rules from Section 2.3.

(No need to do any ϵ - δ stuff, just compute the limit. Please show your work though.)

(Of course, you are welcome to use the derivative rules we learned to *check* your answer.)

Part 3 (5 points)

Remember that $f(x) = x^2$ was defined in Part 2 of this problem, and $f'(x)$ was computed. Find the equation of the tangent line to the graph of $y = f(x)$ at 1.

Part 4 (5 points)

Remember that $f(x) = x^2$ was defined in Part 2 of this problem, and $f'(x)$ was computed.

For which real numbers c is it true that the tangent line to $y = f(x)$ at c goes through the point $(2, 3)$?

(Hint: keeping c as an unknown, find the equation for the tangent line at c . Plug in $x = 2$ and $y = 3$ and deduce the possible values for c .)

Problem 2 (40 points)

Part 1 (10 points)

Find the derivative of the function $f(x) = 3x^5 + 7x - 8$.

Part 2 (10 points)

Find the derivative of the function $f(x) = (2x^2 + 5x)(3x^2 - 1)$.

Part 3 (10 points)

Find the derivative of the function $f(x) = \frac{x^2 - 1}{5x + 3}$.

Part 4 (10 points)

Find the derivative of the function $f(x) = (x^2 + 1)^{17}$.

Problem 3 (30 points)

Suppose that at time $x = 0$, a ball is thrown straight up in the air with a velocity of 64 ft/sec. Taking into account gravity, and the fact that the ball started at 80 ft above the ground, the height of the ball x seconds after being thrown is equal to

$$f(x) = -16x^2 + 64x + 80$$

Part 1 (10 points)

At what time does the ball hit the ground?

Part 2 (10 points)

What is the velocity of the ball at the time that it hits the ground?

Part 3 (5 points)

What direction is the ball moving in at time $x = 2.5$?

Circle one (though please still show your work or thought process):

up

not moving

down

Part 4 (5 points)

What is the maximum height the ball reaches?

(Hint: remember, the ball reaches its maximum height at the exact moment it stops going up and starts going down. What does that mean about the velocity at that moment?)