

Math 13100 – Section 58 – Midterm 2

February 10, 2016

Information and Directions

- This exam will last 50 minutes.
- This is a closed-book exam.
- No electronic devices are allowed to be used during this exam.
- Partial credit is given for showing your calculations and explaining your thoughts.

Name: _____

Problem	Points	Score
1	30	
2	20	
3	25	
4	25	

Problem 1 (30 points)

Part 1 (10 points)

Let c be a real number, and let f be a function. Give the precise definition of what it means to say

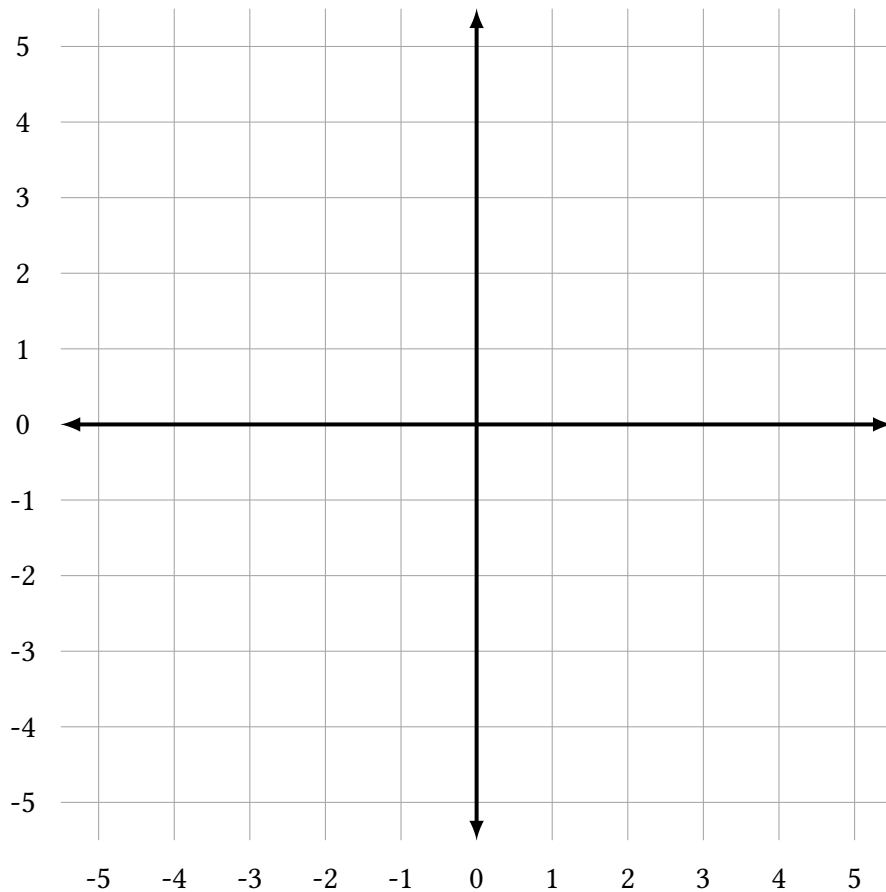
“ f is continuous at c ”

(Hint: there are three conditions that must be met.)

Part 2 (5 points)

Let $f(x) = \frac{x^2 - 3x}{x}$. On this depiction of the plane, draw the graph of the equation $y = f(x)$.

For at least three different points on your graph, label the point with its x - and y -coordinates.



Part 3 (10 points)

Let $f(x) = \frac{x^2 - 3x}{x}$, as in Part 2.

Are there any discontinuities of f ? (That is, can you find any c where f is **not** continuous at c ?) Explain your answer.

Part 4 (5 points)

If f does have any discontinuities, then for each discontinuity, say whether it is removable or not. Explain your answer.

Problem 2 (20 points)

Part 1 (10 points)

Let c be a real number, and let f be a function.

Give the precise, mathematical meaning of the statement

$$\lim_{x \rightarrow c} f(x) = \infty$$

(Hint: your answer shouldn't contain the letter ϵ .)

Part 2 (10 points)

Give an example of a function f such that $\lim_{x \rightarrow 2} f(x) = \infty$.

(Hint: remember, “equals infinity” is not the same thing as “does not exist”.)

Problem 3 (25 points)

Consider the function

$$f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ x - 1 & \text{if } x < 0 \end{cases}$$

Thus, for example,

$$f(0) = 0^2 = 0$$

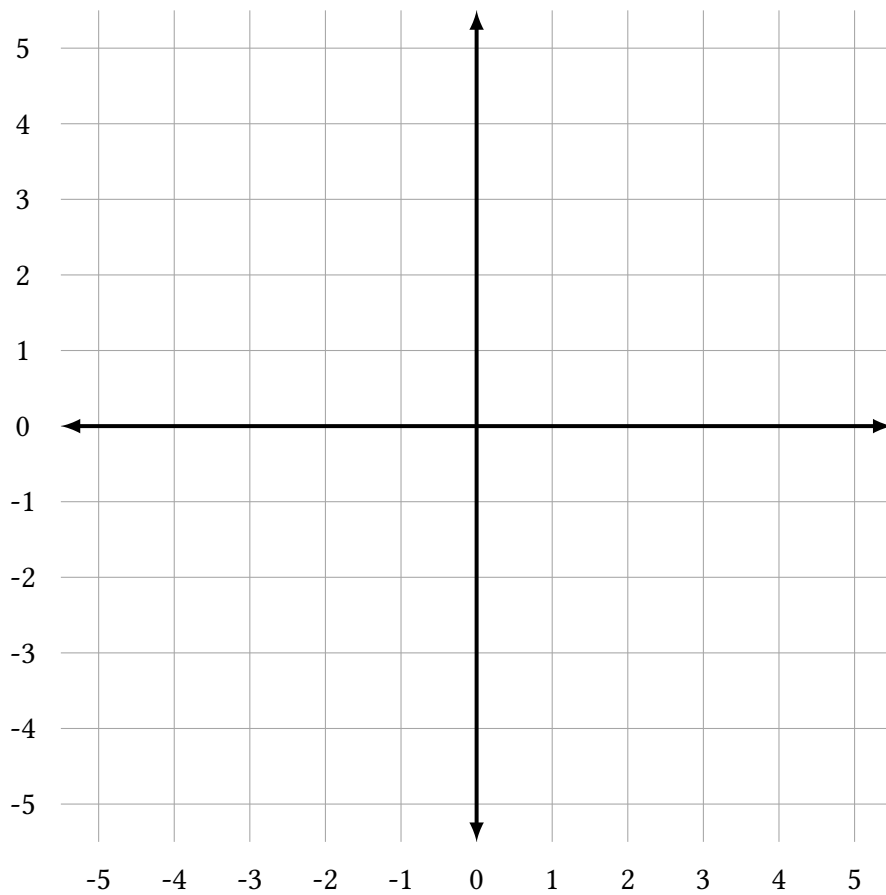
$$f(2) = 2^2 = 4$$

$$f(-2) = (-2) - 1 = -3$$

Part 1 (5 points)

On this depiction of the plane, draw the graph of the equation $y = f(x)$.

For at least three different points on your graph, label the point with its x - and y -coordinates.



Part 2 (10 points)

Remember that $f(x)$ was defined in Part 1 of this problem.

What are the values of these one-sided limits?

$$\lim_{x \rightarrow 0^+} f(x) = \underline{\hspace{2cm}} \qquad \lim_{x \rightarrow 0^-} f(x) = \underline{\hspace{2cm}}$$

You aren't required to give any formal proof for these answers, though any manner of explanation for your answers that you provide below can be helpful in case you end up needing partial credit.

Part 3 (10 points)

Remember that $f(x)$ was defined in Part 1 of this problem.

Does the two-sided limit $\lim_{x \rightarrow 0} f(x)$ exist? Justify your answer.

(Hint: for this question, a great strategy would be to cite your answers in Part 2 and appeal to a theorem from the book that relates one-sided limits to the existence of a two-sided limit.)

Problem 4 (25 points)

Part 1 (10 points)

Let L be a real number, and let f be a function.

Give the precise, mathematical meaning of the statement

$$\lim_{x \rightarrow \infty} f(x) = L$$

(Hint: your answer shouldn't contain the letter δ .)

Part 2 (5 points)

What is the value of $\lim_{x \rightarrow \infty} \frac{3x^2}{9x^2 - 2x + 5}$? You don't have to justify your answer here.

Part 3 (10 points)

Give a proof that $\lim_{x \rightarrow \infty} \frac{3x^2}{9x^2 - 2x + 5}$ is equal to your answer in Part 2.

Of course you are always welcome to create formal proof from scratch, but it would probably be easier to appeal to certain theorem(s) from the book about how limits and operations can be manipulated. Please show individual steps and justify why you are allowed to do what you're doing.