$(a,b) = \{\text{real numbers } x \text{ that satisfy } a < x < b\}$ open

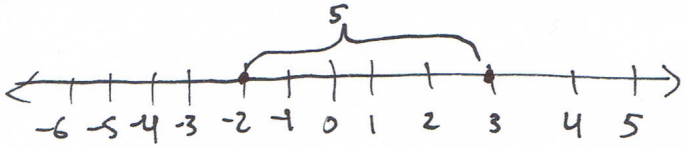
$(a,b] = \{\text{real numbers } x \text{ that satisfy } a < x \leq b\}$ half-open

$[a,b) = \{\text{real numbers } x \text{ that satisfy } a \leq x < b\}$ half-open

$[a,b] = \{\text{real numbers } x \text{ that satisfy } a \leq x \leq b\}$ closed

$|x| =$ x if x itself is positive, and $-x$ if x itself is negative

$|x-y| =$ distance between the real numbers x and y



distance between -2 and 3 is $|(-2)-3| = |-5| = 5$

"the distance between x and c is less than δ"

$\Leftrightarrow$ "$|x-c| < \delta$"

$\Leftrightarrow$ "$c-\delta < x < c+\delta$"

$\Leftrightarrow$ "x is in the open interval $(c-\delta, c+\delta)$"

thus, the solutions to $|x-5| < 3$ are the same as the solutions to $5-3 < x < 5+3$

$2 < x < 8$

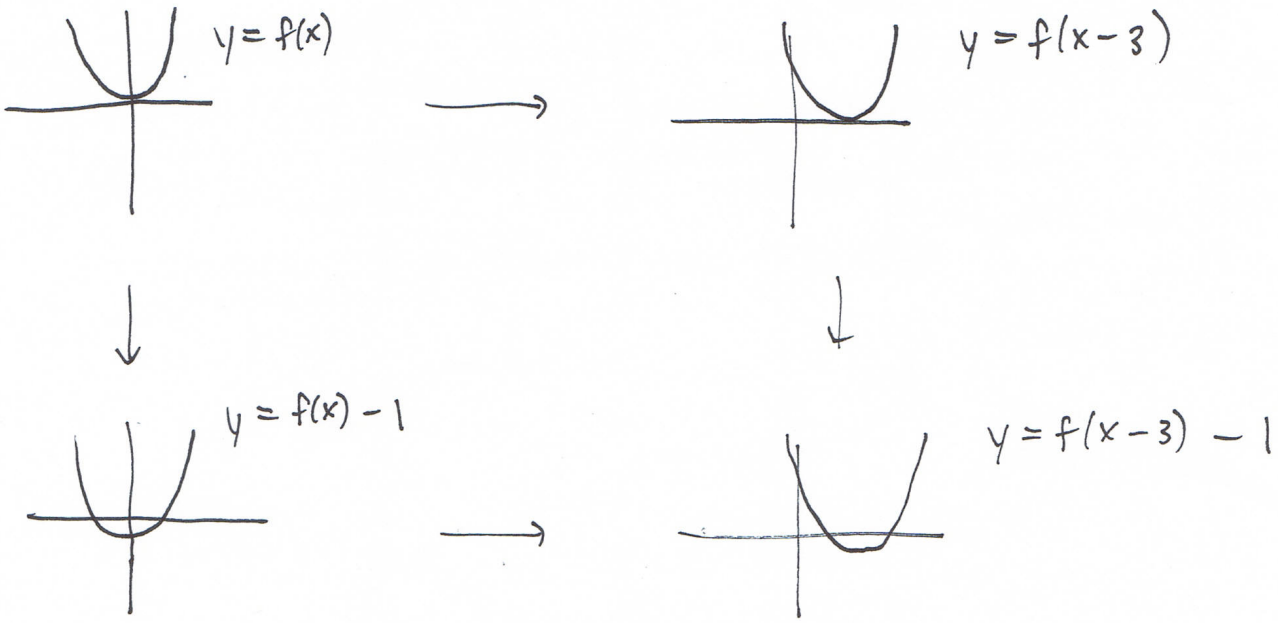
the solutions to $2 < x < 4$ are the same as the solutions to $|x-3| < 1$

$3-1 < x < 3+1$

Shifting "input" affects graph horizontally

Shifting "output" affects graph vertically

Example: $f(x) = x^2$



Note that since $f(x) = x^2$, $f(x-3) = (x-3)^2 = x^2 - 6x + 9$

$\sqrt{x}$ is defined for any x such that $x \geq 0$ (0 allowed!)

$\frac{1}{x}$ is defined for any x such that $x \neq 0$

$(f+g)(x)$ is defined to be equal to $f(x) + g(x)$

$(f \cdot g)(x)$ is defined to be equal to $f(x) \cdot g(x)$

$(f \circ g)(x)$ is defined to be equal to $f(g(x))$

If $f(x) = x^2 - 1$ and $g = 2x$, then $(f+g)(3) = 8 + 6 = 14$ $(f \cdot g)(3) = 8 \cdot 6 = 48$

$(f \circ g)(3) = f(6) = 36 - 1 = 35$

$\lim_{x \to c}$ depends locally on c — i.e., on a neighborhood of c

- does not depend on what $f(c)$ is (or even if it's defined)
- does not depend on what $f(x)$ is for x "far away" from c

This is what Theorem C on p. 70 means

"$\lim_{x \to \infty} \frac{x^2}{2x^2 - 5} = \frac{1}{2}$" and "$\lim_{x \to 3} \frac{x}{x-3}$ does not exist" and so on

are mathematical sentences. You should know

- what sentences like this mean
- how to prove rigorously whether they are true or false

For example,

"$\lim_{x \to \infty} f(x) = L$" means "For any $\varepsilon > 0$, there is some $M > 0$ such that

$$x > M \implies |f(x) - L| < \varepsilon,$$

i.e. $L - \varepsilon < f(x) < L + \varepsilon$ for any $x > M$."

"$\lim_{x \to c} f(x) = -\infty$" means "For any $N > 0$, there is some $\delta > 0$ such that

$$0 < |x - c| < \delta \implies f(x) < -N,$$

i.e. $f(x) < -N$ for any x between $c - \delta$ and $c + \delta$ other than c itself"

"f is continuous at c" means

- $f(c)$ is defined
- $\lim_{x \to c} f(x)$ exists
- $\lim_{x \to c} f(x) = f(c)$

If f is not continuous at c, then we say f has a discontinuity at c.

If f has a discontinuity at c and $\lim_{x \to c} f(x)$ exists, then changing the value of $f(c)$ to be $\lim_{x \to c} f(x)$ makes f continuous at c, we say the discontinuity was removable.

Remember

Main Limit Theorem	p. 68
Substitution Theorem	p. 69
Squeeze Theorem	p. 72
Polynomials and rational functions are continuous wherever they are defined	p. 83
Composite Limit Theorem	p. 85

Intermediate Value Theorem p. 87