

Math 19620 – Section 50 – Midterm 2

November 17, 2015

Information and Directions

- This exam will last 80 minutes.
- This is a closed-book exam.
- No electronic devices are allowed to be used during this exam.
- Make sure to show all details of your work.

Name: _____

Problem	Points	Score
1	25	
2	20	
3	20	
4	15	
5	20	

Problem 1 (25 points)

Consider this 3×5 matrix:

$$A = \begin{bmatrix} 1 & 2 & 1 & 5 & 3 \\ 4 & 9 & 0 & 17 & 9 \\ 2 & 5 & -2 & 7 & 3 \end{bmatrix}$$

Part 1 (10 points)

Find a basis of the kernel of A .

Part 2 (10 points)

Find a basis of the image of A .

Part 4 (5 points)

Complete the statement of the rank-nullity theorem: for any $m \times n$ matrix B , we have

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

(fill in with mathematical expressions)

Use your work in this problem to find the specific values of each of these expressions for the matrix A . Make sure the integers you write below line up exactly with the corresponding expression above.

$$\underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

(fill in with integers)

Problem 2 (20 points)

Consider this 3×3 matrix and this basis of $\mathbb{R}^3$:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & -3 \\ 2 & 7 & -5 \end{bmatrix} \quad \mathcal{B} = \left\{ \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \right\}$$

Part 1 (15 points)

Express the matrix A in the basis $\mathcal{B}$.

Part 2 (5 points)

Let $\vec{w} = 5\vec{v}_1 + 7\vec{v}_2 + 3\vec{v}_3$, so that $[\vec{w}]_{\mathcal{B}} = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$.

Express the vector $A\vec{w}$ in the basis $\mathcal{B}$, i.e. compute $[A\vec{w}]_{\mathcal{B}}$.

(You can do this directly, by computing $\vec{w}$, then computing $A\vec{w}$, then finding that vector's expression in the basis $\mathcal{B}$. There is also a much faster way using your answer from Part 1. Either is acceptable.)

Problem 3 (20 points)

Let V be this subspace of $\mathbb{R}^4$:

$$V = \text{span} \left(\begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 3 \\ 4 \end{bmatrix} \right)$$

Part 1 (10 points)

Find an orthonormal basis of V .

Part 2 (10 points)

Find the matrix of $\text{proj}_V : \mathbb{R}^4 \rightarrow \mathbb{R}^4$.

Problem 4 (15 points)**Part 1 (10 points)**

Compute the value of $\det(A)$, where

$$A = \begin{bmatrix} 3 & 6 & 3 \\ 4 & 10 & 0 \\ 2 & 0 & 8 \end{bmatrix}$$

Part 2 (5 points)

Compute the value of $\det(B)$, where

$$B = \begin{bmatrix} 3 & 6 & 3 \\ 4 & 10 & 0 \\ 14 & 0 & 56 \end{bmatrix}$$

Note that B is the same as A , except that one of its rows has been multiplied by 7.

Problem 5 (20 points)

Consider the system of linear equations $A\vec{x} = \vec{b}$, where

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \\ 2 & 1 & 0 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} -1 \\ 1 \\ -3 \\ 3 \end{bmatrix}$$

Part 1 (15 points)

Find the least-squares solutions of $A\vec{x} = \vec{b}$.

Part 2 (5 points)

Find the vector in $\text{im}(A)$ that is closest to $\vec{b}$.

(For extra credit up to an additional 5 points, give an intuitive explanation of the answer to this part.)