

Exercise 1. Construct an isomorphism of topological rings between the ring of p -adic integers $\mathbf{Z}_p$ and $\mathbf{Z}[[x]]/(x - p)$.

Exercise 2. Let $P \in \mathbf{Z}_p[x]$ be a monic polynomial $P = x^n + a_1 x^{n-1} + \cdots + a_n$ such that $\bar{P} = x^n + \bar{a}_1 x^{n-1} + \cdots + \bar{a}_n \in \mathbf{F}_p[x]$, $\bar{a}_i$ being the image of a_i in $\mathbf{F}_p$, has a simple zero $\bar{\alpha} \in \mathbf{F}_p$. Prove that there exists P has a zero $\alpha \in \mathbf{F}_p$ of which image in $\mathbf{F}_p$ is $\bar{\alpha}$.

Exercise 3. Answer the following questions and justify your answers.

1. Is $\prod_{p \in \mathcal{P}} p\mathbf{Z}_p$ an open subset of $\mathbf{A}_{\text{fin}}$? Is it a closed subset? Is it compact?
2. Is $\mathbf{A}_{\text{fin}} \cap \prod_{p \in \mathcal{P}} p^{-1}\mathbf{Z}_p$ an open subset of $\mathbf{A}_{\text{fin}}$? Is it a closed subset? Is it compact?