Q: When can you continuously associate a point on each fiber?

\[ \text{Do you have a splitting?} \]

The first example is universal bundle:

\[ 1 \to \pi_1(S_g) \to \text{Mod}_g \xrightarrow{\phi} \text{Mod}_g \to 1 \]

Fact: No! finite subgroup argument.

The main theorem today is:

Today will talk about another universal bundle:

\[ \text{Diff}^+(S_{g,n}) : \text{Diff gp with fixing n pts individually.} \]

\[ \text{PMod}_n = \text{To (Diff}^+(S_{g,n})) \]

\[ S_g \to \mathcal{H}_{g,n} \]

\[ \mathcal{H}_{g,n} \]

\[ K(\text{PMod}_n \times 1) \]

Q: Does this bundle have section?
R. Han conjecture: A section is homotopic to one of them.

Man Theorem (C.) \( g \geq 2 \ n \geq 1 \)

A section of the universal bundle \( U_{g,n} \) is homotopic to \( S_i \) for some \( i \).

\[ \xymatrix{ \text{PConf}_n(S_g) \ar[r] & \text{Tri} (\text{PConf}_n(S_g)) \ar[r]^P & \text{PMod}_n \ar[r]_{\text{Tri}} & \text{Mod}_g \ar[r] & 1 } \]

\( \text{PConf}_n(S_g) \) in distinct points in \( S_g \)

\( \text{PConf}_n(S_g) \xrightarrow{\sim} S_g \times S_g \times \ldots \times S_g \)

\( \text{Tri} (\text{PConf}_n(S_g)) \cong \text{PBr}_n(S_g) \)

Diagram:

- Coloring
- Backboard

PB is the PBn(Sg)
Classification theorem (CC)

Let $R : \text{BBn}(S_g) \to \pi_1(S_g)$ has to be

1. $\text{Im} R \cong \mathbb{Z}$
2. $R$ factor through $\Phi : \pi_1(S_g) \to \Phi_1(S_g)$

How cohomology plays the role, in here?

Goal: Proof of CT

1. $\phi : \text{PCafn} \to S_g^n$

Lemma: $\phi$ induces a homomorphism on $H^1(\mathbb{R})$ coefficient

$H^1(\text{PCafn}) = H^1(S_g^n) = \bigoplus H^1(S_g) \cong \bigoplus \Phi_i H^1(S_g)$

We'll call $A_i$ the $i$th copy of $H^1(S_g)$ i.e. $A_i = \Phi_i H^1(S_g)$.

$\phi : S_g \to \bigoplus \text{PCafn}(S_g)$

Now apply SSS to fibration & induction.

The main pt is to understand how $\Phi_1(S_g)$ acts on $H^1(S_g \times S_g)$.
Lemma 2*: (A property of cup product $H^\wedge H^\prime \to H^\prime$ of $PB_n(S)$)

\[ \forall x, y \in H^\prime(Pract(S)) \quad \text{if } x \cup y = 0 \]

Then $\exists i, x, y \in i^*H^\prime(S)$ $\implies$ (Harder Induction $\& SSS$, Thurston)

Final moment: pf of CT

$\text{Im}R \leq \pi_1(S)$

Given a hom $R: PB_n(S) \to \text{Im}R = \mathbb{F}n$

pick $a \in H^i(\mathbb{F}n) \ni a^x = 0$

$R^*(a) \cup R^*(x) = 0 \implies R^*a \in i^*H^\prime(S)$

i.e., $R^*a = -i^*b$

Goal now: Go back to section problem:

$\text{K to } PB_n(S) \xrightarrow{\phi} \pi_1(S)$

trivial $\downarrow R \xrightarrow{\phi} \mathbb{F}n \xrightarrow{a} \mathbb{Q}$

Fact* fig normal subgp of $\mathbb{F}n$ is either trivial or finite index

$K \equiv PB_n(S; -pt)$

$H^i(PB_n(S; -pt)) \cong \oplus H^i(S; )$ by diagonal.