Extensions of Conway's Game of Life

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Introduction

Background

Conway's Game of Life is a cellular automaton invented by the British mathematician John Horton Conway in 1970 [3]. The universe of the Game of Life is an infinite, two-dimensional orthogonal grid of square cells, each of which is either alive or dead. Every cell interacts with its eight neighbors, which are the cells that are horizontally, vertically, or diagonally adjacent.

Rules

- A living cell with two or three living neighbors survives to the next time step
- A cell with no or one living neighbor dies
- A dead cell with exactly three neighbors will be alive in the next time step

1. Cancer Cell Extension

A Third State:

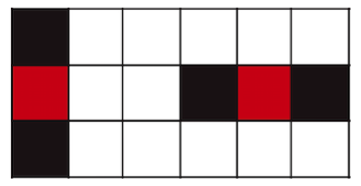
Probabilistic extension can be further studied for many future works[1]. Here we introduce another state into Conway's Game of Life, which is cancer cell. With the third state, the rules will be slightly different than the original rules.

Cancer Rules

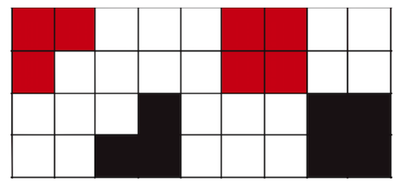
There are three states in total: alive, dead, and cancer.

- An alive cell will be non-dead on the next time step, when there are two or three non-dead neighbors. If there are more alive cells than cancer cells around, it will be alive. Otherwise, it will be cancer.
- A dead cell will become alive when there are three non-dead neighbors. If there are more alive cells than cancer cells around, it will be alive. Otherwise, it will be cancer.
- A cancer cell will always be cancer to the next time step when there are between one and three non-dead neighbors. A cancer cell can never be alive unless it dies and becomes alive again.

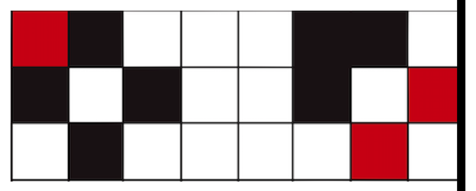
Example of Stable patterns with Cancer Cells



Blinker(Oscillator)



Beacon(Oscillator)



Boat(Still life)

2. Probabilistic Extension

- **Heuristic**: Most Games of Life with random initial conditions become still lives and oscillators [2]
- **Problem**: This doesn't fit biology, which we might try to model with Game of Life
- **Solution**: introduce probabilistic rules to the game of life and look for periodicity in live cell counts over time
- **Analysis**: Use the discrete Fourier transform to analyze periodic behavior. Hopefully find periodicity!

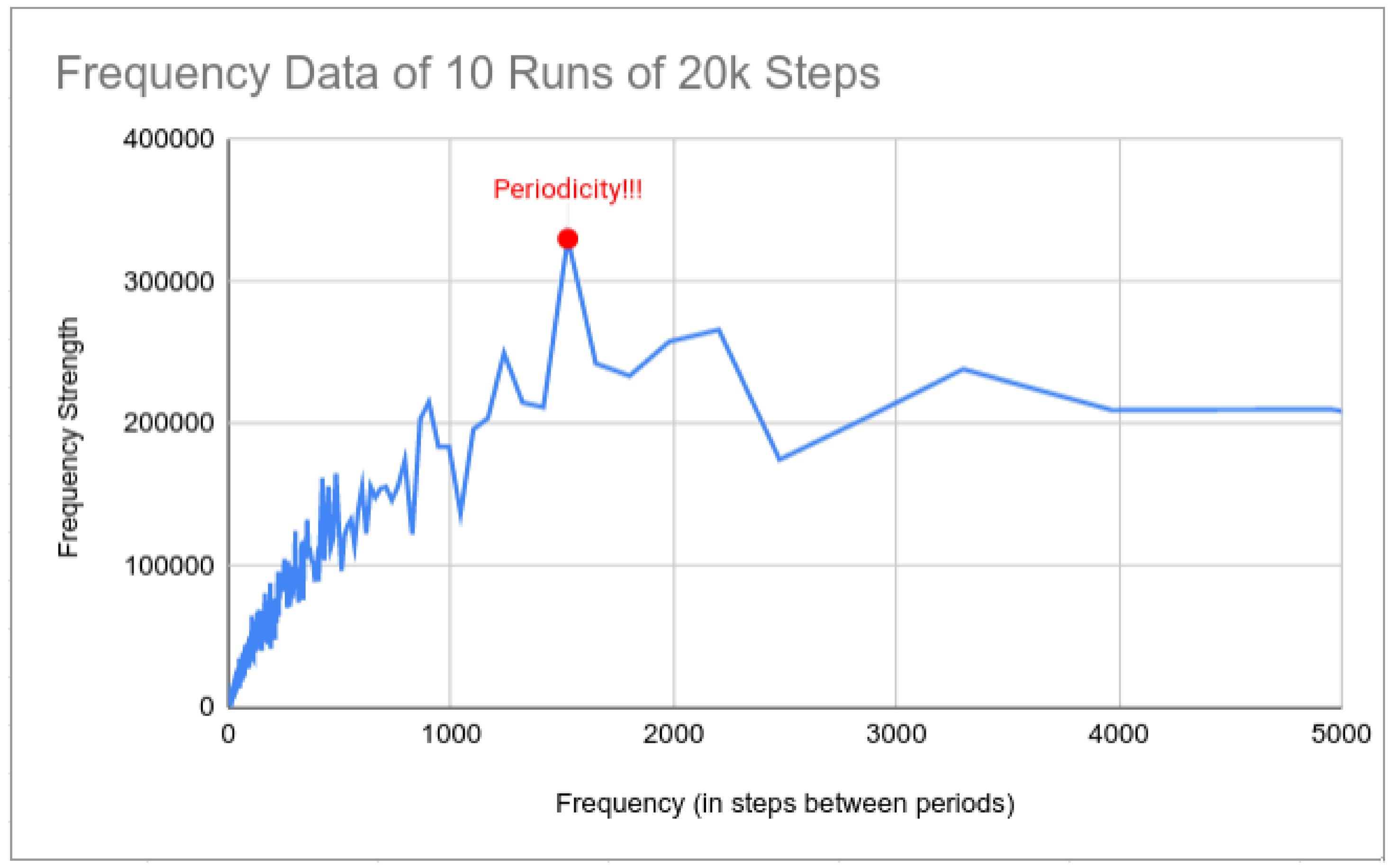
Probabilistic Rules

Our probabilistic rules are specified by numbers $r_0, \ldots, r_8$, $s_0, \ldots, s_8$, $d_0, \ldots d_8$ which represent probabilities of reproduction, "staying," and death, respectively. We require that $r_i + s_i + d_i = 1$ for each $0 \leq i \leq 8$, and the transition function is

- At each time step, a cell with i neighbors has probability r_i of being alive at the next step regardless of its current state, probability s_i of staying at its current state, and probability d_i of being dead at the next step regardless of its current state.

Conway's traditional game of life is given by $r_3 = 1$, and every other $r_- = 0$, $s_2 = 1$ with every other $s_- = 0$, and $d_i = 1$ for $i \neq 2, 3$. Our specific rule-set starts with these probabilities, and changes them so that $s_2 = 0.99$ and $r_2 = 0.1$.

Here's the Fourier transform data we got out!



3. Weighted Layers Extension

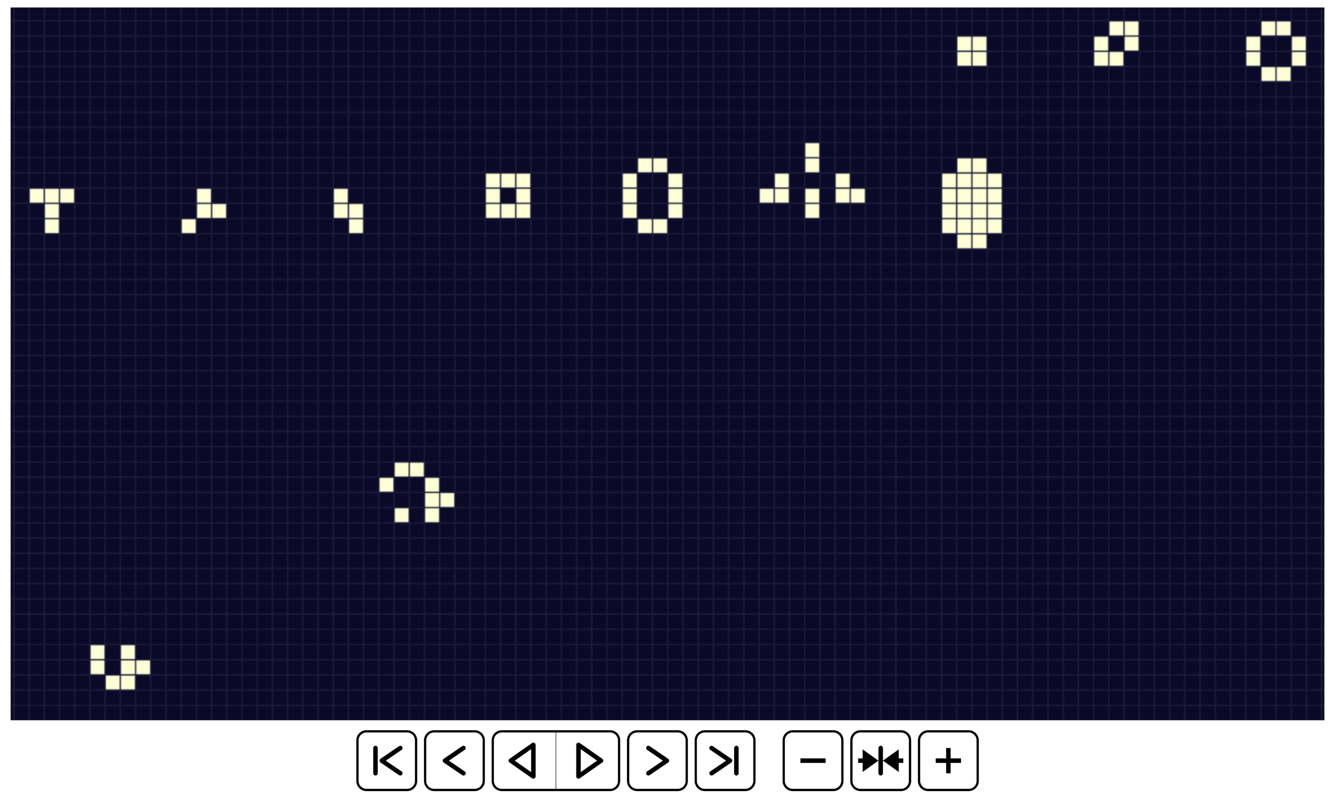
Weighted Layers Rules

Conway's original rule takes into consideration of each cell's immediate 8 neighbors. In this extension we consider more layers, inspired by Newton's universal law of gravitation $F = GMm/r^2$, we weight the number of alive neighbor cells in the r-th layer around the cell by $1/r^2$.

$$\text{\# of live neighbors} = \sum_r \frac{1}{r^2} \cdot (\text{\# of live neighbors in } r\text{-th layer})$$

We then have rules for the transition function with specific numbers to exhibit nice Game of Life style behavior.

Example of patterns



Future Directions

The cancer cell game could be further refined in order to model real cancer, and we can search for gliders in this game. The extension with more layers has many new kinds of patterns that are not in Conway's original game, and a glider that has sense of spin, which will be exciting to be explored in three dimensions. Also, the Fourier Analysis method could be used on other probabilistic extensions as well as the other extensions in order to see if they display periodicity [4].

References

[1] Gabriel Aguilera-Venegas, José Luis Galán-García, Rocío Egea-Guerrero, María Á. Galán-García, Pedro Rodríguez-Cielos, Yolanda Padilla-Domínguez, and María Galán-Luque. A probabilistic extension to Conway's Game of Life. *Adv. Comput. Math.*, 45(4):2111–2121, 2019.

[2] Matthew Cook. Still life theory. 12 2000.

[3] Martin Gardner. Mathematical games the fantastic combinations of john conway's new solitaire game "life". *Scientific American*, 223:120–123, Oct 1970.

[4] Pierre-Yves Louis and Francesca R. Nardi. *Probabilistic Cellular Automata*, volume 27 of *Emergence, Complexity and Computation ECC*. Springer, 2018.