Announcements

- First office hours tonight, 8pm-9pm, use the "Zoom Lounge"
- Homework #1 due 8pm Friday on Gradescope.

Definition .0.1

We need a few definitions for working with CW complexes!

- A CW-complex is called finite if it involves a finite number of cells.
- A subcomplex of a CW complex is a closed subset consisting of a union of cells.

Exercise .0.1

A subcomplex is itself a CW complex.

.0.1. Operations on CW Complexes

Definition .0.2

We can consider the product of two CW complexes can be given a CW complex structure.

Namely, given X and Y CW complexes, we can take two cells e_α^n from X and e_β^m from Y we can form the product space $e_\alpha^n \times e_\beta^m$ which is homeomorphic to an $(n+m)$-cell. We take these products as the cells for $X \times Y$

Warning It is possible (in "pathological" cases) that the product topology on $X \times Y$ does not agree with the weak topology. They do agree if either X or Y is locally compact or if X and Y have at most countably many cells

Exercise .0.2

The torus is $S^1 \times S^1$. Write down the CW complex structure on the torus that comes from the CW complex structure on S^1 with one point and one edge.

Definition .0.3

If X is a CW complex and A is a subcomplex, then the quotient X/A (A is identified to a point) inherits a CW complex structure. Namely

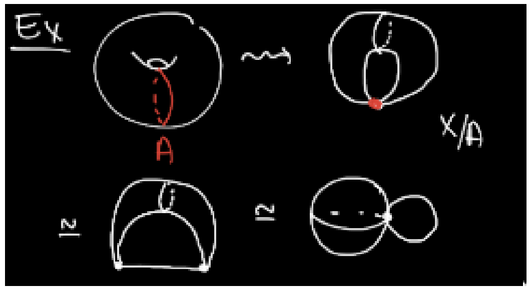
- The 0-skeleton is points in $X^0 - A^0$ unioned with one point for A
- Each cell in $X^n - A$ is attached to $(X/A)^n$ by the attaching map defined by composing with the quotient map $S^n \to X^n \to X^n/A^n$.

Example .0.3

We can take the sphere and squish the equator down to form a wedge of two spheres as follows:

Example .0.4

We can take the torus and squish down a ring around the hole like in this picture:



The above is homotopy equivalent to a 2-sphere wedged with a 1-sphere via the extending the red point into a line, and then sliding the left point of the line along the two-sphere towards the other point, forming a circle.

Definition .0.4

Take X, Y to be CW complexes, and let $x_0 \in X^0, y_0 \in Y^0$. Then we can consider $X \vee Y$ which is the quotient of $X \cup Y$ by identifying x_0 and y_0 to one point, called the wedge sum.