Announcements

- Midterm II - 1 week from tomorrow!
  - Practice package available this weekend
  - Let me know about conflicts ASAP
  - Covers material up to / including this Friday + Homework 8 (focus on material since Midterm I)
  - Study groups will be set up with When2Meet
- Extra OH next week (7-9pm on Mar 17)
- Corrections to Homework #7 Problem #6 posted

Back to Math!

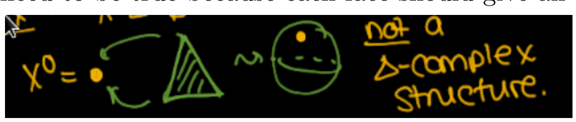
Definition .0.1
The boundary $\partial\Delta^n$ of Δ^n is the union of its faces. The open simplex $\mathring{\Delta}^n$ is $\Delta^n \setminus \partial\Delta^n$.

Definition .0.2 (Δ-complex)
A Δ-complex structure on X is a collection of maps $\sigma_\alpha : \Delta^n \to X$ (n depends on α) such that:

(i) $\sigma_\alpha\big|_{\mathring{\Delta}^n}$ is injective, and each point in X is in the image of exactly one such restriction.
(ii) Each restriction of σ_α to a face of Δ^n must coincide with a map $\sigma_\beta : \Delta^{n-1} \to X$.
(iii) A set A in X is open if and only if $\sigma_\alpha^{-1}(A)$ is open in Δ^n for all α. (i.e., X is the quotient space $\coprod_{n,\alpha} \Delta^n \to X$)

Exercise .0.1
The Δ-complex structure is a CW-complex structure. But with condition that attaching maps must be injective on the interior of each face individually (must glue faces onto existing simplices).

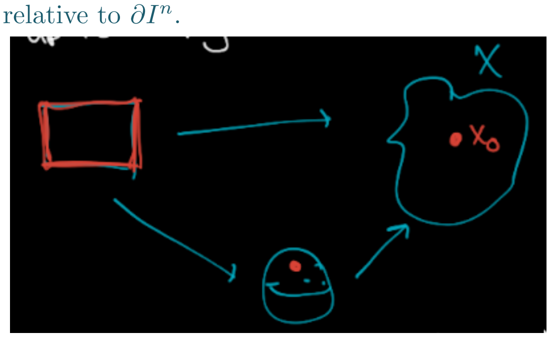
Non-Example .0.2
Take $X = S^2$. A CW Complex structure can be a 0-skeleton of a point, and then glue on a 2-cell by mapping the entire boundary to a single point. This is not injective on each of the faces of the triangle given below (which would need to be true because each face should give an attaching map). Nice!



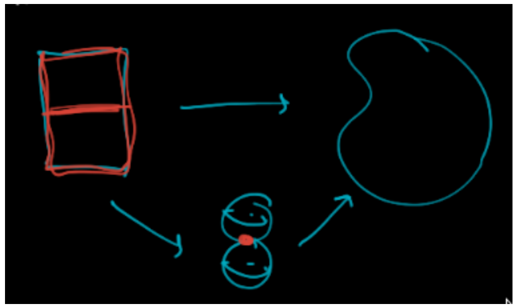
There is a Δ-complex structure on S^2, but this particular structure doesn't work.

.1. Motivation for Homology

Definition .1.1 (Informal Higher Homotopy Groups)
Define π_n to be homotopy classes of based maps from $I^n \to (X, x_0)$ which maps the boundary to the basepoint, up to homotopy relative to ∂I^n.



We get a group structure (and even nicer for $n \geq 2$ it's abelian!!!)



Problem: Although these are the natural, and very useful, they are **really** hard to compute. So hard that the higher homotopy groups of the k-sphere $\pi_n(S^k)$ is an open question for $n \geq k$.

Instead, we will study homology groups. They are much easier to compute–however their definition is a bit less intuitive.