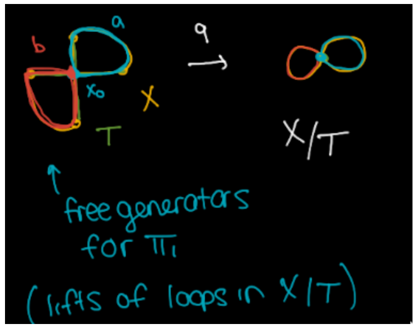
The current program is to calculate the fundamental groups of CW complexes. For now, we need to see that the fundamental group of a 1-skeleton (a graph) can be found by taking a maximal tree, and then quotienting the space by that tree to get a wedge of circles:



Proof (Maximal Trees Exist). Recall X is a quotient of $X^0 \coprod_\alpha I_\alpha$. Each subset U is open if and only if it intersects each edge $\overline{e_\alpha}$ in an open subset. A map $X \to Y$ if and only if its restriction to each edge $\overline{e}_\alpha$ is continuous.

Take X_0 to be a subgraph.

Goal: Construct a subgraph Y with

- $X_0 \subseteq Y \subseteq X$
- Y deformation retracts to X_0
- Y contains all vertices of X.

So if we take X_0 to be a vertex, then Y is our tree and we're done!

Strategy: Build sequence $X_0 \subseteq X_1 \subseteq \cdots$ and corresponding $Y_0 \subseteq Y_1 \subseteq \cdots$. We start with X_0 and inductively define:

$$X_i = X_{i-1} \bigcup \text{ all edges } \overline{e_\alpha} \text{ with one or both vertices in } X_{i-1}$$

Exercise .0.1

Check that $X = \bigcup_i X_i$. In Hatcher we do this by arguing the union on the right is both open and closed.

Now let $Y_0 = X_0$. By induction, we will assume that Y_i is a subgraph of X_i such that:

- Y_i contains all vertices of X_i
- Y_i deformation retracts to Y_{i-1}

We can then construct Y_{i+1} by taking Y_i and adding to it one edge to adjoin every vertex of X_{i+1}:

$$Y_{i+1} = Y_i \bigcup \text{ one edge to adjoint every vertex of } X_i$$

This is possible by using the axiom of choice.

Exercise .0.2

Check that Y_{i+1} deformation retracts to Y_i (just smush down each edge).

Exercise .0.3

Y deformation retracts to $Y_0 = X_0$ by performing the deformation retraction fromY_i to Y_{i-1} during the time interval $[1/2^i, 1/2^{i-1}]$

Awesome! We win!