Announcements

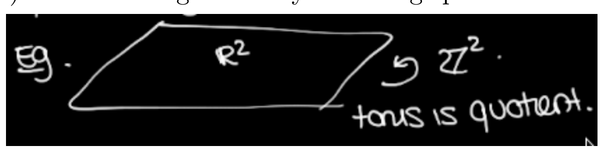
- Corrections to Homework #7
- Quiz on Wednesday on covering spaces
  - Know definition of action of the fundamental group on a fiber
  - Know the definition of a regular cover
  - Know the result on Deck Transformation group of regular cover
  - Know the action of $N(p_*(\pi_1(\widetilde{X}, \widetilde{x})))$ by Deck transformations.
- Midterm II - 1 week from Thursday

.1. Covering Space Recap so far

- Lifting Properties
  - Homotopy lifting property (extending $\widetilde{F}_0$, existence / uniqueness)
  - Path lifting (existence / uniqueness given a preimage of $\gamma(0)$)
  - Lifts of $f : X \to Y$ (given conditions on $f_*(\pi_1(X))$ in $\pi_1(Y)$.
- Classification given by:

$$\{\text{basepoint-preserving isomorphism classes of covers } p : (\widetilde{X}, \widetilde{x}_0) \to (X, x_0)\} \to \{\text{Subgroups of } \pi_1(X, x_0)\}$$
$$p \mapsto p_*(\pi_1(\widetilde{X}, \widetilde{x}_0))$$

  - Constructed the universal cover of X (corresponds to trivial subgroup of $\pi_1(X)$ whenever X is path connected, locally path connected, and simply connected
  - Constructed cover X_H corresponding to a subgroup H
  - Proved uniqueness up to isomorphism
- Deck Transformations
  - Classified regular covers using normal subgroups of $\pi_1(X, x_0)$
  - Showed that $G(\widetilde{X}) \cong N(H)/H$
- Next (current homework): Constructing covers by "covering space actions"

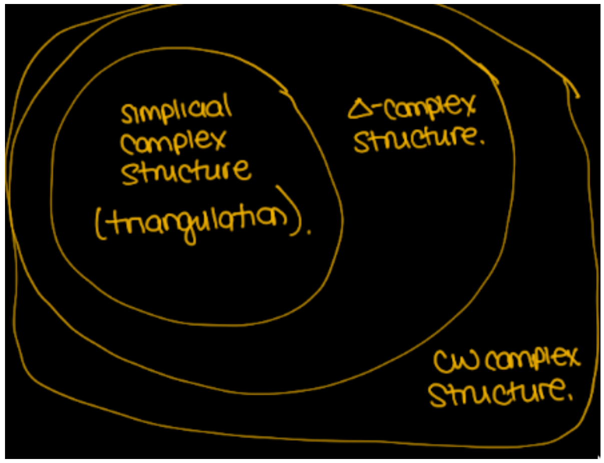


Then we can view covers as quotients by group actions that satisfy the properties of a "covering space action"

I. Homology

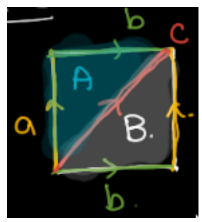
I.1. Δ-complexes

This is a stricter version of a CW complex which allows us to decompose our spaces into cells. In terms of how things fit together, we have this diagram:

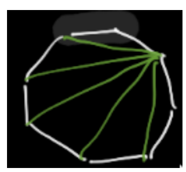


Example I.1.1

The torus with the following edges a, b, c, and gluing in triangles A and B



For this delta complex notice we've glued down a triangle whose vertices are all identified, this is not allowed in a simplicial complex / triangulation. We can also do it for genus 2 surfaces:



Definition I.1.1 (Simplices)

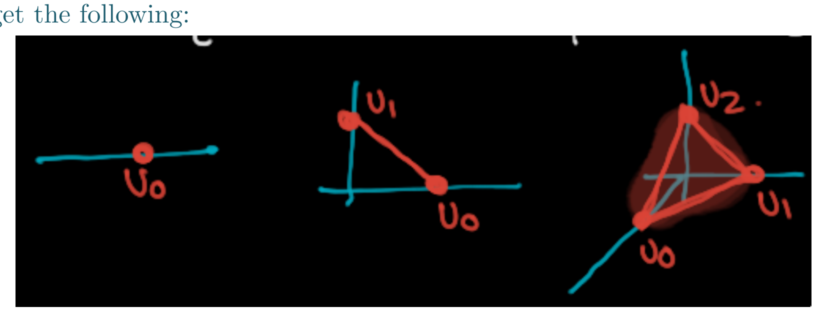
A 0-simplex is a point. A 1-simplex is an interval. A 2-simplex is a triangle. A 3-simplex is a tetrahedron. . . so what's a simplex?

Well, in general, a n-simplex is always the convex hull of $(n+1)$ points in $\mathbb{R}^n$. We can view simplices as both combinatorial and topological objects.

The standard n-simplex is given by:

$$\Delta^n = \{(t_0, \dots, t_n) \in \mathbb{R}^{n+1} \mid \sum t_i = 1, t_i \geq 0 \ \forall i\}$$

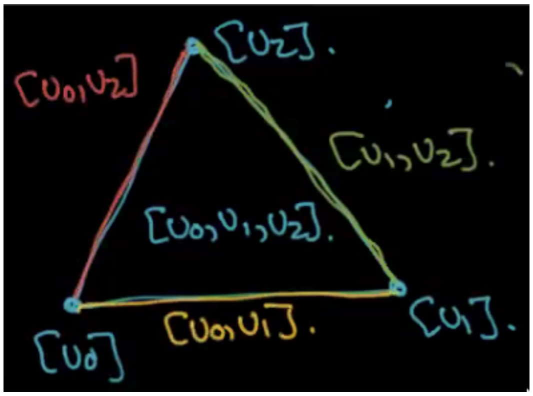
In pictures we get the following:



Our simplices will implcitly come with a choice of ordering of the vertices as $\Delta^n = [v_0, \dots, v_n]$ (the convex hull of these points with this ordering).

Definition I.1.2

A face of a simplex $\Delta^n = [v_0, \dots, v_n]$ is a subsimplex spanned by any n of the $n+1$ vertices with the induced order.



Exercise I.1.2

In general, any subset of k vertices in Δ^n spans a subsimplex of dimension $k-1$.

The order on the vertices of Δ^n also induces an order on the vertices of every subsimplex.

Definition I.1.3

A subsimplex of Δ^n is:

- Combinatorially, a subset of the vertices
- Topologically, we can identify with a smaller dimensional simplex

A face is a subsimplex of 1 dimension lower than Δ^n ("codimension 1").