REU Talks

Thursday, August 9, 2012

Alex Becker: *Billiards in Convex Polygons, and Why the Game of Pool is Really Played on a Torus*

This talk concerns the behavior of a billiard ball bouncing around inside a convex polygon, which is a typical example of a dynamical system. In particular, we will examine the existence and properties of periodic trajectories in this system. We will see how trajectories are classified and analyzed, particularly in the case of polygons with angles equal to rational multiples of pi, for trajectories correspond to paths on certain Riemann surfaces. As advertised in the title, in the case of a rectangle the Riemann surface in question is a torus. If time permits, we will analyze how this approach fails in when the angles are not rational multiples of pi and see what we can still say about these cases.

Kerisha Burke: *Exploring Graphs of Triangulated n-gons: Connectivity & Finding an Upper Bound on the Diameter*

Our research uses graph theory to understand the geometry of the space for triangulated n-gons with one interior vertex. The theory and applications of graphs are the basis of many strides in the field of science and mathematics. A quite notable graph theory problem, resolved by Sleator-Tarjan-Thurston, serves as the motivation for this research. In their findings, Sleator-Tarjan-Thurston explored the connection between the maximum rotation distance of binary trees and the graph $R_{n,0}$, a graph with triangulated n-gons as nodes. As an extension of the works by Sleator-Tarjan-Thurston, we study the graph $R_{n,1}$, for $n \geq 3$. Given the graph $R_{n,1}$ such that its nodes are triangulated n-gons with one interior vertex, this study explores the connectivity and bounds on the diameter. By constructing the graph $R_{n,1}$ and studying its properties, we proved that the nodes are connected by a sequence of flipped edges. Lastly, we found an algorithm to compute the distance between any of the nodes of $R_{n,1}$ and a special vertex in $R_{n,1}$ that is connected to all nodes. The proof of the diameter bounds is a corollary of the algorithm.

Alexander Dunlap: *Basic Percolation Theory*

Percolation is the study of connectedness in randomly-chosen subsets of an infinite graph, subject to a certain *density* parameter. We will describe the surprising phenomenon of criticality in percolation, in which the percolation system's behavior changes abruptly at a *critical* value of the density. In discussing a few short proofs used to establish the existence of the critical density, we will illustrate several aspects of percolation systems that are useful for understanding and working with them.

Erika Dunn-Weiss: *Manifolds of Constant Curvature*

This talk will be an introduction to the concept of curvature on a manifold. We will develop an intuition for curvature and discuss the model spaces of constant curvature for Riemannian manifolds (and semi-Riemannian manifolds if we have the time!) We will also introduce Gauss's Theorema Egregium and a few classification theorems for these spaces.

Alex Grant: *Spaces of Negative Curvature and Preissman's Theorem*

From high school geometry, we should all know what spaces of negative curvature look like: the textbook saddle-shaped drawing of the hyperbolic plane. The question is: what do *compact* spaces of negative curvature look like? If you want the answer to that question, this talk is not for you. I will, instead, be discussing Preissman's Theorem, an important result concerning the fundamental group of such spaces.