General Information: The first Mini-Midterm will start at the beginning of class on Thursday, October 16. Here is some information about the structure of the exam. It will be 50 minutes long and consist of 3 questions: you will be able to choose 3 out of a possible 4. Of these four, at least one question will be from the homework and at least one will be from the review sheet. There may be an optional challenge problem of small but nontrivial weight. The exam is worth 12.5% of the total course grade. (The remaining 30 minutes of class will be a lecture covering new material.)

The exam will cover material up to and including the lecture on Tuesday, October 14 and the homework due Wednesday, October 15.

I intend for this exam to simply make sure everyone has a good understanding of the basic set theory we’ve covered so far, as this will be used throughout the rest of the course.

Review: The last 30 minutes of class on Tuesday, October 14 will be a brief review. I’ll also hold extra office hours Tues, Oct 14 from 3-4pm.

Major Topics and Results: You should be able to give the definition, give examples and non-examples, prove the result, state and prove the theorem, or use the technique, as appropriate.

- Ordinals. The class Ord of ordinals is well-ordered by $\epsilon$. Every well-ordered set is isomorphic to a unique ordinal.
- Cardinals. The Cantor-Schroeder-Bernstein theorem. Assuming AC, every set has a well defined cardinality. The definitions of addition and multiplication of cardinals. The fundamental theorem of cardinal arithmetic.
- Transfinite induction and recursion.

Review Problems are on the back.
Review Problems. *I suggest you try these without notes for best results.*

(1) Prove that a strictly increasing function $f : \mathbb{R} \to \mathbb{R}$ cannot have more than countably many discontinuities.

(2) Prove that every ordinal $\alpha$ has a unique successor, which we denote $\alpha + 1$.

(3) Sketch a proof that $\epsilon$ is a linear order on Ord.

(4) Assuming that $\epsilon$ is a linear order on Ord, prove it is a well order.

(5) Give an example of a set which is well-ordered by $\epsilon$ but is not an ordinal.

(6) Prove that the union of a set of ordinals is an ordinal and the union of a set of cardinals is a cardinal.

(7) Prove that the intersection of a set of ordinals is an ordinal. What about the intersection of a set of cardinals?

(8) Prove that $\aleph_\omega = \bigcup_{n<\omega} \aleph_n$. First explain what is to be proved.

(9) Let $\alpha$ be an ordinal and let $\kappa = \aleph_{\alpha+1}$. Show that $\kappa$ is not the sum of fewer than $\kappa$ cardinals each of which has size strictly less than $\kappa$.

(10) Show that the previous problem is false when $\kappa = \aleph_\omega$.

(11) State and prove the principle of transfinite induction.

(12) Prove that $\mathbb{R}^3$ can be written as the disjoint union of circles of radius 5734.

(13) Prove the real plane cannot be covered with fewer than continuum many lines.

(14) Prove there exists a subset of $\mathbb{R}^2$ with exactly two points on every line.