1. Suppose $A, B \subset \mathbb{Q}$ and $A \cup B$ is open. Must $A$ and $B$ both be open? Must at least one of $A$ and $B$ be open?

2. Give an example of a collection of regions in $\mathbb{Q}$ whose intersection is a single point in $\mathbb{Q}$.

3. Let $X$ be a topological space and let $A \subset X$. We say that a collection $\mathcal{U}$ of subsets of $X$ is an open cover of $A$ if

   - Each $U \in \mathcal{U}$ is open; and
   - $A \subset \bigcup_{U \in \mathcal{U}} U$.

   If $\mathcal{V} \subset \mathcal{U}$ is also an open cover of $A$, we say $\mathcal{V}$ is a subcover of $\mathcal{U}$. Finally, if $\mathcal{V}$ is a subcover of $\mathcal{U}$ and $\mathcal{V}$ is also a finite set, we say that $\mathcal{V}$ is a finite subcover of $\mathcal{U}$.

   Consider the topological space $\mathbb{Q}$ and the subsets $A$, $B$, and $C$ of $\mathbb{Q}$ defined by

   $$A := \{ n^{-1} \mid n \in \mathbb{N} \}, \quad B := \{ q \in \mathbb{Q} \mid 0 < q < 1 \}, \quad \text{and} \quad C := A \cup \{0\}.$$  

   (a) Give an example of an open cover of $A$ which has no finite subcover.

   (b) Give an example of an open cover of $B$ which has no finite subcover.

   (c) Prove that every open cover of $C$ has a finite subcover.

4. Corollary 3.20 of Script #3 stipulates that $G$ must be nonempty. Is this stipulation necessary? Is it still true if we remove this hypothesis? Explain.