Problem Set # 3 / MATH 16100

Due: Tuesday, November 10, 2015, at 11:50 CST

Remark: We use the notation $A \subseteq B$ to mean $A \subset B$ and $A \neq B$.

1. Prove directly (i.e., without appealing to Theorem 2.24) that $C$ is an infinite set.

2. For each of the following, either
   - Prove that, independent of the realization of $C$, there exists a nonempty set $A \subset C$ such that the given set is nonempty; or
   - Prove that there is a realization of $C$ such that there exists a nonempty set $A \subset C$ such that the given set is nonempty, but there is also another realization of $C$ such that, for every set $A \subset C$, the given set is empty.

   (a) $A \cap LP(A)$;
   (b) $(C \setminus A) \cap (C \setminus LP(A))$;
   (c) $A \setminus LP(A)$;
   (d) $LP(A) \setminus A$.

3. For each of the following, either
   - Prove that, independent of the realization of $C$, there exists a nonempty set $A \subset C$ such that the given statement is true; or
   - Prove that there is a realization of $C$ such that there exists a nonempty set $A \subset C$ such that the given statement is true, but there is also another realization of $C$ such that, for every nonempty set $A \subset C$, the given statement is not true.

   (a) $A \subseteq LP(A)$;
   (b) $LP(A) \subseteq A$;
   (c) $A \cap LP(A) = \emptyset$;
   (d) $A = LP(A)$.

4. Prove or disprove: if $A \subset C$, then $LP(LP(A)) \subset LP(A)$.