1. (a) (3 points) Define what it means for a relation $\sim$ on a set $S$ to be an equivalence relation. \textit{(You do not have to say what ‘relation’ means).}

(b) (2 points) If $S$ is a set and $\sim$ is an equivalence relation on $S$, define the equivalence class $C(a)$ of an element $a \in S$.

(c) (3 points) Prove that, if $a, b \in S$ with $a \sim b$, then $C(a) = C(b)$.

(d) (2 points) Let $S$ be the power set of $\{1, 2, 3, 4, 5, 6\}$, and let $\sim$ be the equivalence relation on $S$ defined by: $A \sim B$ if the sum of the elements of $A$ is equal to the sum of the elements of $B$. Write down the equivalence class $C(\{1, 5\})$ of $\{1, 5\} \in S$ (no proof necessary).

2. (a) (4 points) Prove that, if $f : A \rightarrow B$ and $g : B \rightarrow C$ are injective, then so is $g \circ f$.

(b) (4 points) Prove that, if $f : A \rightarrow B$ and $g : B \rightarrow C$ are functions such that $g \circ f$ is injective and $f$ is surjective, then $g$ is injective.

(c) (2 points) Give an example of a function $f : \mathbb{Z} \rightarrow \mathbb{Z}$ that is surjective but not injective.

3. (a) (4 points) Prove from the ring axioms that, if $R$ is a ring, $0$ is the additive identity of $R$, and $a \in R$, then $0 \times a = 0$.

(b) (4 points) Let $R$ be the set $\mathbb{Z}^2 = \{(a, b)|a, b \in \mathbb{Z}\}$ with the following two operations:

$$
(a, b) + (c, d) = (a + c, b + d)
$$

$$
(a, b) \times (c, d) = (ac - bd, ad + bc).
$$

Prove that $\times$ is associative — in other words, show that axiom MA holds.

(c) (2 points) In fact, these operations make $R$ into a ring, with additive identity $(0, 0)$, multiplicative identity $(1, 0)$, and $-(a, b) = (-a, -b)$.

Write down an element of $R$ whose square is $(-1, 0)$.