Homework 2
Math 15300 (section 51), Spring 2015

This homework is due in class on Wednesday, April 15th. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.

You are encouraged to think about the problems marked with a (*), but they are not to be handed in.

0. (*) (a) Read sections 8.2-8.5 in the text.
   (b) Review $u$-substitution from last term. You will have trouble on this problem set (and the rest of the course!) if you are not completely comfortable with it.

1. Calculate:

   (a) (Ex 8.2.2) $\int_0^2 x^2 dx$
   (b) (Ex 8.2.3) $\int x^2 e^{x^3} dx$
   (c) (Ex 8.2.4) $\int x^3 e^{-2x^2} dx$
   (d) (Ex 8.2.7) $\int \frac{x^2}{\sqrt{1-x}} dx$
   (e) (Ex 8.2.9) $\int \frac{e^{x^2}}{x} dx$
   (f) (Ex 8.2.13) $\int (\ln x)^2 dx$
   (g) (Ex 8.2.20) $\int_{\pi/2}^0 x^2 \sin x dx$
   (h) (Ex 8.2.22) $\int x^2 (2x - 1)^{-7} dx$
   (i) (Ex 8.2.23) $\int e^x \sin x dx$
   (j) (Ex 8.2.25) $\int_0^1 \ln(x^2 + 1) dx$
   (k) (Ex 8.2.34) $\int \cos \sqrt{x} dx$ (Hint: Let $u = \sqrt{x}$, $dv = \frac{\cos \sqrt{x}}{\sqrt{x}} dx$)
   (l) (Ex 8.2.38) $\int \cos(\ln x) dx$ (Hint: Use integration by parts twice.)

2. Calculate:

   (a) (Ex 8.3.2) $\int_{\pi/8}^0 \cos^2 (4x) dx$
   (b) (Ex 8.3.5) $\int \cos^4 x \sin^3 x dx$
   (c) (Ex 8.3.8) $\int \sin^2 x \cos^4 x dx$
   (d) (Ex 8.3.11) $\int \tan^3 x dx$
   (e) (Ex 8.3.30) $\int \frac{\sin^3 x}{\cos x} dx$
   (f) (Ex 8.3.43) $\int_0^{\pi/6} \tan^2 2x dx$

3. Calculate:

   (a) (Ex 8.4.3) $\int \sqrt{x^2 - 1} dx$
   (b) (Ex 8.4.4) $\int \frac{x}{\sqrt{4-x^2}} dx$
   (c) (Ex 8.4.8) $\int \frac{x^2}{\sqrt{4+x^2}} dx$
   (d) (Ex 8.4.13) $\int_0^5 x^2 \sqrt{25-x^2} dx$
   (e) (Ex 8.4.24) $\int \frac{dx}{e^x \sqrt{4+e^{2x}}}$
   (f) (Ex 8.4.33) $\int \frac{x}{(x^2 + 2x + 5)^2} dx
4. Derive the formulas:
   (a) (Ex 8.2.42) \( \int \arctan x \, dx = x \arctan x - \frac{1}{2} \ln(1 + x^2) + C \)
   (b) (Ex 8.2.43) \( \int x^k \ln x \, dx = \frac{x^{k+1}}{k+1} \ln x - \frac{x^{k+1}}{(k+1)^2} + C \)

5. Use identity (8.4.3) (on page 421) to calculate the following:
   (a) (Ex 8.4.38) \( \int \frac{dx}{(1 + x^2)^2} \)
   (b) (Ex 8.4.39) \( \int \frac{dx}{(1 + x^2)^3} \)

6. (Ex 8.2.73a) As you can probably guess, if you were to integrate \( \int x^3 e^x \, dx \) by parts, you would get
   \[ \int x^3 e^x \, dx = (Ax^3 + Bx^2 + Cx + D)e^x + E \]
   for some constants \( A, B, C, D \) and \( E \). Differentiate the above expression to find \( A, B, C \) without integrating.

7. Recall the identity \( f(x) = f(0) + \int_0^x f'(t) \, dt \) from last term. Assume that all \( n^{th} \) derivatives of \( f \) exist, and are continuous.
   (a) Use integration by parts to show \( f(x) = f(0) + f'(0)x + \int_0^x (x-t)f''(t) \, dt \) (Hint: Let \( u = f'(t) \) and \( v = x - t \), so that \( dv = -dt \).)
   (b) Use integration by parts again to show that \( f(x) = f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \int_0^x \frac{(x-t)^2}{2}f'''(t) \, dt \).
   (c) (*) Keep going. What happens after \( n \) steps? We will need this idea later in the term, when we talk about Taylor series.

8. (Ex 8.3.53)
   (a) Use integration by parts to show that for \( n > 2 \)
   \[ \int \sin^n x \, dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, dx. \]
   (Hint: Let \( u = \sin^{n-1} x \), \( dv = \sin x \, dx \). Use the fact that \( \int \cos^2 x \sin^{n-2} x \, dx = \int \sin^{n-2} x \, dx - \int \sin^n x \, dx \), as \( \cos^2 x = 1 - \sin^2 x \).)
   (b) Deduce that \( \int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x \, dx. \)
   (c) (*) Use induction to prove that for \( n \geq 2 \)
   \[ \int_0^{\pi/2} \sin^n x \, dx = \begin{cases} \frac{(n-1) \cdots 3 \cdot 1}{n \cdots 6 \cdot 4 \cdot 2} \frac{\pi}{2} & \text{n is even} \\ \frac{(n-1) \cdots 4 \cdot 2}{n \cdots 5 \cdot 3} & \text{n is odd} \end{cases} \]
   (d) (*) Show that \( \int_0^{\pi/2} \sin^n x \, dx = \int_0^{\pi/2} \cos^n x \, dx. \)
   (e) Use parts (c) and (d) to compute \( \int_0^{\pi/2} \sin^7 x \, dx \) and \( \int_0^{\pi/2} \cos^6 x \, dx. \)