This homework is due in class on Wednesday, January 28th. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.

You are encouraged to think about the problems marked with a (*), but they are not to be handed in.

0. (*) Read sections 5.4-5.5 in the text.

1. Evaluate the following integrals (Hint: the table on page 144 may be helpful for some of these):

   (a) (Ex 5.4.2) \[ \int_0^1 (3x + 2) \, dx \]  
   (b) (Ex 5.4.6) \[ \int_0^4 \sqrt{x} \, dx \]  
   (c) (Ex 5.4.8) \[ \int_1^2 \left( \frac{3}{x^3} + 5x \right) \, dx \]  
   (d) (Ex 5.4.17) \[ \int_0^1 (x + 1)^7 \, dx \]  
   (e) (Ex 5.4.23) \[ \int_0^{\pi/2} \cos x \, dx \]  
   (f) (Ex 5.4.26) \[ \int_{\pi/6}^{\pi/3} \sec x \tan x \, dx \]  
   (g) (Ex 5.4.26) \[ \int_{\pi/4}^{\pi/2} \csc x (\cot x - 3 \csc x) \, dx \]  
   (h) (Ex 5.4.33) \[ \int_0^4 \frac{d}{dx} \left[ \sqrt{4 + x^2} \right] \, dx \]  
   (i) (Ex 5.4.57) \[ \int_{-\pi/2}^{\pi} f(x) \, dx \], where \[ f(x) = \begin{cases} 1 + 2 \cos x & -\pi/2 \leq x \leq \pi/3 \\ (3/\pi)x + 1 & \pi/3 < x \leq \pi. \end{cases} \]

2. Sketch the region bounded by the two curves, and find its area:

   (a) (Ex 5.5.12) \( y = 6x - x^2 \), \( y = 2x \)  
   (b) (Ex 5.5.16) \( y = \sqrt{x} \), \( y = \frac{1}{4} x \)  
   (c) (Ex 5.5.23) \( y = \cos x \), \( y = 4x^2 - \pi^2 \).

3. Find the critical points for \( F \) and, at each critical point, determine whether \( F \) has a local minimum, a local maximum, or neither:

   (a) (Ex 5.3.17) \( F(x) = \int_0^x \frac{t - 1}{1 + t^2} \, dt \)  
   (b) (Ex 5.3.18) \( F(x) = \int_0^x \frac{t - 4}{1 + t^2} \, dt \)
4. (Ex 5.3.20) For \( x > 0 \), set \( F(x) = \int_0^x t(t - 3)^2 \, dt \).

(a) Find the critical points of \( F \), and determine the intervals on which \( F \) increases, and the intervals on which \( F \) decreases.

(b) Determine the concavity of \( F \), and find the points of inflection, if any.

(c) Sketch the graph of \( F \).

5. (Ex 5.4.40) Define a function \( F \) on \([0, 4]\) such that \( F'(x) = \sqrt{1 + x^2} \) and

(a) \( F(3) = 0 \)  
(b) \( F(3) = 1 \).

6. (Ex 5.4.62) Let \( f \) be a function such that \( f'(x) \) is continuous on \([a, b]\). Show that

\[
\int_a^b f'(t)f(t) \, dt = \frac{f(b)^2 - f(a)^2}{2}
\]

by finding an antiderivative for \( f'(t)f(t) \).

7. Given that \( f \) is a continuous function, let \( F(x) = \int_0^x xf(t) \, dt \). Find \( F'(x) \). (Hint: The answer is \( NOT \, xf(x) \).)