This homework is due in class on Wednesday, January 14th. You may cite results from class as appropriate. Unless otherwise stated, you must provide a complete explanation for your solutions, not simply an answer. You are encouraged to work together on these problems, but you must write up your solutions independently.

You are encouraged to think about problems marked with a (*), but they are not to be handed in.

0. (*) Read sections 4.6-4.8 in the text.

1. In each of the problems below, sketch a graph of the given function. Draw any horizontal or vertical asymptotes (if any). Be sure that your graph includes all of the relevant features of the graph (e.g. $x$ and $y$-intercepts, critical points, inflection points, concavity, increasing and decreasing intervals, etc.). For this question you do not need to give any explanation beyond an answer.

   (a) (Ex 4.8.4) $f(x) = x^3 - 9x^2 + 24x - 7$
   (b) (Ex 4.8.9) $f(x) = x^2 + \frac{2}{x}$
   (c) (Ex 4.8.29) $f(x) = x + \sin 2x$
   (d) (Ex 4.8.42) $f(x) = x^2(x - 7)^{1/3}$
   (e) (Ex 4.8.50) $f(x) = 3 + 2 \cot x + \csc^2 x, x \in \left(0, \frac{\pi}{2}\right)$

2. (Ex 4.6.36) Sketch the graph of a continuous function $f$ satisfying all of the given conditions. For this question you do not need to give any explanation beyond an answer:
   - $f''(x) > 0$ if $|x| > 2$, $f''(x) < 0$ if $|x| < 2$
   - $f'(0) = 0$, $f'(x) > 0$ if $x < 0$, $f'(x) < 0$ if $x > 0$
   - $f(0) = 1$, $f(-2) = f(2) = \frac{1}{2}$, $f(x) > 0$ for all $x$
   - $f$ is an even function.

3. Describe the concavity of graph of the following functions, and find the points of inflection (if any):

   (a) (Ex 4.6.6) $f(x) = x + \frac{1}{x}$
   (b) (Ex 4.6.8) $f(x) = 2x^2 - 5x + 2$
   (c) (Ex 4.6.10) $f(x) = x^3(1 - x)$
   (d) (Ex 4.6.12) $f(x) = \frac{x + 2}{x - 2}$
   (e) (Ex 4.6.19) $f(x) = \sin^2 x, x \in [0, \pi]$
   (f) (Ex 4.6.21) $f(x) = x^2 + \sin 2x, x \in [0, \pi]$
4. Find the vertical and horizontal asymptotes of the following functions:

(a) (Ex 4.7.3) \( f(x) = \frac{x}{3x-1} \)  
(b) (Ex 4.7.6) \( f(x) = \frac{4x}{x^2+1} \)
(c) (Ex 4.7.10) \( f(x) = \frac{4x^2}{(3x-1)^2} \)  
(d) (Ex 4.7.17) \( f(x) = \sqrt{x+4} - \sqrt{x} \)

5. Determine whether the graph of \( f \) has a vertical tangent or a vertical cusp (or neither) at the point \( x = c \):

(a) (Ex 4.7.22) \( f(x) = 3 + x^{2/5}; c = 0 \)  
(b) (Ex 4.7.28) \( f(x) = 4 - (2 - x)^{3/7}; c = 2 \)

6. (*) (Ex 4.8.47) Let \( p \) and \( q \) be positive integers, \( q \) odd, \( p < q \). Let \( f(x) = x^{p/q} \). Specify conditions on \( p \) and \( q \) so that:

(a) The graph of \( f \) has a vertical tangent at \((0,0)\).
(b) The graph of \( f \) has a vertical cusp at \((0,0)\).

7. (Oblique Asymptotes) We say that the line \( y = ax + b \) is an Oblique Asymptote for \( f(x) \) if \( f(x) - (ax+b) \to 0 \) as either \( x \to \infty \) or \( x \to -\infty \) (i.e. the graph of \( f \) is approaching the line \( ax+b \)).

(a) Show that the line \( y = (b/a)x \) is an oblique asymptote to the hyperbola \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) in the first quadrant. (Hint: Write \( \sqrt{x^2 - a^2} - x = \frac{-a^2}{x + \sqrt{x^2 - a^2}} \) to evaluate the limit).

(b) (Ex 4.7.50) Find all vertical and oblique asymptotes of \( f(x) = \frac{2x^2 + 3x - 2}{x + 1} \). Sketch a graph of \( f \).

8. (Ex 4.8.57) Let \( f(x) = \frac{x^3 - x^{1/3}}{x} \). Show that \( f(x) - x^2 \to 0 \) as \( x \to \pm \infty \). This means that \( f \) is asymptotic to the parabola \( y = x^2 \). Sketch a graph of \( f \) illustrating this.

9. (Ex 4.6.3) The three graphs on page 194 of the textbook show: the graph of a function \( f \), the graph of \( f' \), and the graph of \( f'' \) but NOT in the proper order. Determine which curve is the graph of which function. Explain your reasoning.