A. Török has pointed out to us the the need for a better proof of Theorem B. Accordingly, the first two full paragraphs on page 539 should be replaced with the following argument.

We are trying to show that the subfoliation of the center unstable leaves by the strong unstable leaves is of class C^1. Let W denote the disjoint union of the center unstable leaves,

$$W = \bigsqcup W^{cu}(p).$$

It is a non-separable manifold of class C^1. For partial hyperbolicity implies that its tangent bundle $TW = E^{cu}$ is continuous. The restriction of TM to W is a C^1 bundle $T_W M$ that contains the C^0 subbundle TW. Since f is a diffeomorphism of class C^2, the tangent map

$$Tf : T_W M \to T_W M$$

is a C^1 bundle isomorphism. As in the proof of Theorem A, approximate E^u, E^{cs} by smooth bundles $\widetilde{E}^u$, $\widetilde{E}^{cs}$ and express Tf with respect to the splitting $TM = \widetilde{E}^u \oplus \widetilde{E}^{cs}$ as

$$\begin{pmatrix} A & B \\ C & K \end{pmatrix}.$$

Let $\widetilde{\mathcal{P}}(1)$ be the bundle over W whose fiber at p is the set of linear maps $P : \widetilde{E}^u_p \to \widetilde{E}^{cs}_p$ such that $\|P\| \leq 1$. The linear graph transform sends P to

$$\Gamma_{Tf}(P) = (C + KP) \circ (A + BP)^{-1}.$$

It is a bundle map that covers f on W, contracts fibers by approximately $\|K\|\|A^{-1}\| \doteq \|T^c f\|/m(T^u f)$, and contracts the base at worst by approximately $m(A) \doteq m(T^c f)$. The unique invariant section $p \mapsto P_p$ of $\widetilde{\mathcal{P}}(1)$ of Γ_{Tf} has graph$P_p = E^u_p$. Center bunching implies that

$$\text{(fiber contraction)(base contraction)}^{-1} \doteq \frac{\|T^c f\|}{m(T^u f)}(m(T^c f))^{-1} < 1,$$

so fiber contraction dominates base contraction and the invariant section is of class C^1. That is, E^u is a C^1 bundle over the C^1 manifold W. Since E^u is tangent to the foliation $\mathcal{W}^u$, it is integrable.

Frobenius' Theorem states that the foliation tangent to a C^k integrable subbundle of TW is of class C^k, in the sense that there is a C^k atlas of foliation charts covering the manifold W. Strictly speaking, the proof requires that the underlying manifold is of class C^{k+1}, so we need to re-check the result in the case of the C^1 manifold W.

Locally, $W^{cu}(p)$ is the graph of a C^1 function $g : E_p^{cu} \to E_p^s$. The linear projection $\pi : E_p^{cu} \times E_p^s \to E_p^{cu}$ restricts to a C^1 diffeomorphism $\pi_p : W^{cu}(p) \to E_p^{cu}$,

$$\pi_p : (x, g(x)) \mapsto x.$$

The tangent to π gives a C^1 bundle surjection

$$T\pi : T_{W^{cu}(p)}M \to T(E_p^{cu}).$$

The restriction of $T\pi$ to $E^u|_{W^{cu}(p)}$ agrees with $T\pi_p$, which implies that

$$T\pi_p : E^u|_{W^{cu}(p)} \to T\pi_p(E^u|_{W^{cu}(p)})$$

is a C^1 bundle isomorphism. The latter bundle is C^1 and is integrated by the foliation $\pi_p(\mathcal{W}^u)$. Since $E^{cu}(p)$ is smooth (being a plane), we can apply Frobenius' Theorem to conclude that the foliation $\pi_p(\mathcal{W}^u)$ is C^1. Therefore the foliation $\mathcal{W}^u|_{W^{cu}(p)}$ is also of class C^1.