PROBLEM SET 6, 25600 SECTION 31

Due Friday May 8[1] in class.

1. (10 points) Fraleigh, problem 41-6.
2. (10 points) Let X be the surface of a cube (in $\mathbb{R}^3$). Find a triangulation of X and compute:
a) The Euler characteristic $\chi(X)$.
b) The homology groups $H_0(X)$, $H_1(X)$ and $H_2(X)$.
Verify that your calculations agree with Theorem 43-7 in Fraleigh.
3. (5 points) Prove that $\partial_{n-1} \circ \partial_n = 0$. Hint: For fixed i and j, count how many times $P_1 \cdots \hat{P}_i \cdots \hat{P}_j \cdots P_{n+1}$ occur in $\partial_{n-1}(\partial_n(P_1 \cdots P_{n+1}))$, where $\hat{P}$ means "remove P".
4. (5 points) Fraleigh, problem 42-6.
5. (5 points) Fraleigh, problem 42-7, the even parts.
6. (5 points) Fraleigh, problem 43-6.
7. (10 points) We say that a sequence

$$\ldots \to A_{n+1} \xrightarrow{f_{n+1}} A_n \xrightarrow{f_n} A_{n-1} \xrightarrow{f_{n-1}} A_{n-2} \to \ldots$$

is a *long exact sequence* if the kernel of f_{n-1} is equal to the image of f_n for each n.

Theorem 1. *Given a "nice" inclusion $f : A \to X$ we get a long exact sequence in homology:*

$$\ldots \to H_n(A) \to H_n(X) \to H_n(X/A) \to H_{n-1}(A) \to \ldots,$$

where X/A is the space obtained by collapsing A to a point.

(This sequence is not exact at the very end, but if we replace $H_0(X/A)$ by $\tilde{H}_0(X/A) = ker(H_0(X) \to H_0(pt) = \mathbb{Z})$, then it is.)

Use this theorem to verify that $H_i(S^n) = 0$ for $i \neq 0, n$ and $H_n(S^n) = \mathbb{Z}$, using induction and the cofiber sequence of the nice inclusion $S^{n-1} \to D^n$. (We know that D^n, denoted E^n in the book, is contractible, so $H_i(D^n) = 0$ for $i \neq 0$.)

[1] You can have an extension to Monday May 11 if you want