HOMEWORK 5, 20400 SECTION 51

Due Fri Feb 16 at 5pm in the mailbox marked "Angeltveit (20400) section 51" in the basement.

• Fitzpatrick Section 15.1: 3,5,7,9.

• Fitzpatrick Section 15.2: 1,2,5,6,9.

• We proved in class (or will prove) that if A is an $n \times n$ matrix with $\det A \neq 0$ and B is defined by $b_{ij} = \frac{1}{\det A}(-1)^{i+j} det A^{ji}$ then $(AB)_{ii} = 1$. Finish the proof that B is the inverse of A by showing that for $i \neq j$ we have $(AB)_{ij} = 0$. Hint: express $(AB)_{ij}$ as the determinant of some suitably chosen matrix C you know has determinant 0.

• We proved in class (or will prove) that we can calculate $\det A$ by cofactor expansion along any row, as $\det A = \sum_{j=1}^n (-1)^{i+j} a_{ij} \det A^{ij}$. Prove that $\det A = \det A^T$, and conclude that we can also calculate $\det A$ by cofactor expansion along any column as $\det A = \sum_{i=1}^n (-1)^{i+j} a_{ij} \det A^{ij}$.

For example, you can prove this by induction on n, the case $n = 1$ being trivial and $n = 2$ being easy to check. For $n \geq 3$, write

$$\det A = a_{11} \det A^{11} + \sum_{j=2}^{n} (-1)^{j+1} a_{1j} \det A^{1j}.$$

Now you want to compare this to

$$a_{11} \det A^{11} + \sum_{i=2}^{n} (-1)^{i+1} a_{i1} \det A^{i1}.$$

Notice that the first summand in each expression matches up. Now expand the rest of the summands in the first expression along the first column and the rest of the summands in the second expression along the first row.

• We proved in class (or will prove) that $\det A \neq 0$ if and only if the column vectors of A are linearly independent. Prove that $\det A \neq 0$ if and only if the row vectors of A are linearly independent. (A very simple proof is now possible.)