HOMEWORK 4, 20400 SECTION 51

Due Fri Feb 2 at 5pm in the mailbox marked "Angeltveit (20400) section 51" in the basement.

• Fitzpatrick Section 14.2: 1,3,4,5,9.

• Fitzpatrick Section 14.3: 1,3,7,8,9,11.

• The original version of this problem was wrong. The problem was that the original matrix A was not symmetric, and the theory I was alluding to only works for symmetric matrices. So I'm replacing the original problem with the following:

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, let $v = (1, -1)$ and let $w = (1, 1)$.
a) Calculate Av and Aw, and write the answer in terms of v and w.
b) Show that any $u \in \mathbb{R}^2$ can be written as $u = c_1 v + c_2 w$ for $c_1, c_2 \in \mathbb{R}$ in a unique way.
c) Now calculate Au in terms of c_1, c_2, v and w, and calculate $\langle Au, u\rangle$, again in terms of c_1 and c_2. (Note that $\langle v, w\rangle = 0$.)
d) Let $B = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$. Then B is clearly positive definite. Explain how this shows that A is also positive definite.
e) Now suppose A is a symmetric (meaning $a_{ij} = a_{ji}$) 2×2 matrix, and that there are nonzero vectors v and w with $Av = \lambda_1 v$ and $Aw = \lambda_2 w$, where $\lambda_1, \lambda_2 > 0$ and $\lambda_1 \neq \lambda_2$. Prove that $\langle v, w\rangle = 0$. Hint: view v as a $1 \times n$ matrix and w as an $n \times 1$ matrix, and consider $(vA)w = v(Aw)$. You know that $Aw = \lambda_2 w$. What can you say about vA? And what is vw?

Conclude as in d) that A is positive definite.